2

Electrotechnology

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Contents

Electrotechnology concerns the electrophysical and allied principles applied to practical electrical engineering. A completely general approach is not feasible, and many separate *ad hoc* technologies have been developed using simplified and delimited areas adequate for particular applications.

In establishing a technology it is necessary to consider whether the relevant applications can be dealt with (a) in *macroscopic* terms of physical qualities of materials in bulk (as with metallic conduction or static magnetic fields); or (b) in *microscopic* terms involving the microstructure of materials as an essential feature (as in domain theory); or (c) in molecular, atomic or *subatomic* terms (as in nuclear reaction and semi-conduction). There is no rigid line of demarcation, and certain technologies must cope with two or more such subdivisions at once. Electrotechnology thus tends to become an assembly of more or less discrete (and sometimes apparently unrelated) areas in which methods of treatment differ widely.

To a considerable extent (but not completely), the items of plant with which technical electrical engineering deals—generators, motors, feeders, capacitors, etc.—can be represented by *equivalent circuits* or *networks* energised by an electrical *source*.

For the great majority of cases within the purview of 'heavy electrical engineering' (that is, generation, transmission and utilisation for power purposes, as distinct from telecommunications), a *source* of electrical energy is considered to produce a *current* in a conducting *circuit* by reason of an *electromotive force* acting against a property of the circuit called *impedance*. The behaviour of the circuit is described in terms of the energy fed into the circuit by the source, and the nature of the conversion, dissipation or storage of this energy in the several circuit components.

Electrical phenomena, however, are only in part associated with conducting circuits. The generalised basis is one of magnetic and electrical fields in free space or in material media. The fundamental starting point is the conception contained in Maxwell's electromagnetic equations (Section 1.5.3), and in this respect the voltage and currents in a circuit are only representative of the fundamental field phenomena within a restricted range. Fortunately, this range embraces very nearly the whole of 'heavy' electrical engineering practice. The necessity for a more comprehensive viewpoint makes itself apparent in connection with problems of long-line transmission; and when the technique of ultra-high-frequency work is reached, it is necessary to give up the familiar circuit ideas in favour of a whole-hearted application of field principles.

2.1 Nomenclature

2.1.1 Circuit phenomena

Figure 2.1 shows in a simplified form a hypothetical circuit with a variety of electrical energy sources and a representative selection of devices in which the energy received from the source is converted into other forms, or stored, or both. The forms of variation of the current or voltage are shown in *Figure 2.2*. In an actual circuit the current may change in a quite arbitrary fashion as indicated at (*a*): it may rise or fall, or reverse its direction, depending on chance or control. Such random variation is inconveniently difficult to deal with, and engineers prefer to simplify the conditions as much as possible. For example (*Figure 2.2*(*b*)), the current may be assumed to be rigidly constant, in which case it is termed a *direct current*. If the current be deemed to reverse cyclically according to a sine function, it becomes a *sinusoidal alternating current* (*c*). Less simple waveforms, such as (*d*), may be dealt with by

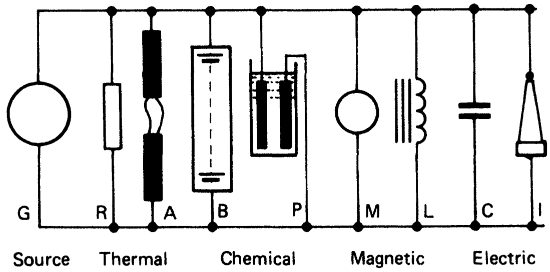


Figure 2.1 Typical circuit devices. G, source generator; R, resistor; A, arc; B, battery; P, plating bath; M, motor; L, inductor; C, capacitor; I, insulator

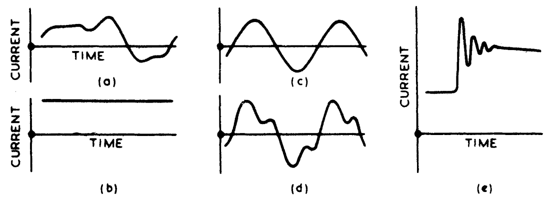


Figure 2.2 Modes of current (or voltage) variation

application of Fourier's theorem, thus making it possible to calculate a great range of practical cases—such as those involving rectifiers—in which the sinusoidal waveform assumption is inapplicable. The cases shown in (*b*), (*c*) and (*d*) are known as *steady states*, the current (or voltage) being assumed established for a considerable time before the circuit is investigated. But since the electric circuit is capable of storing energy, a change in the circuit may alter the conditions so as to cause a redistribution of circuit energy. This occurs with a circulation of *transient* current. An example of a simple oscillatory transient is shown in *Figure 2.2*(*e*).

The calculation of circuits in which direct currents flow is comparatively straightforward. For sine wave alternating current circuits an algebra has been developed by means of which problems can be reduced to a technique very similar to that of d.c. circuits. Where non-sinusoidal waveforms are concerned, the treatment is based on the analysis of the current and voltage waves into fundamental and harmonic sine waves, the standard sine wave method being applied to the fundamental and to each of the harmonics. In the case of transients, a more searching investigation may be necessary, but there are a number of common modes in which transients usually occur, and (so long as the circuit is relatively simple) it may be possible to select the appropriate mode by inspection.

Circuit *parameters*—resistance, inductance and capacitance—may or may not be constant. If they are not, approximation, linearising or step-by-step computation is necessary.

2.1.1.1 Electromotive-force sources

Any device that develops an electromotive force (e.m.f.) capable of sustaining a current in an electric circuit must be associated with some mode of energy conversion into the electrical from some different form. The modes are (1) mechanical/electromagnetic, (2) mechanical/electrostatic, (3) chemical, (4) thermal, and (5) photoelectric.

2.1.2 Electrotechnical terms

The following list includes the chief terms in common use. The symbols and units employed are given in *Table 2.1*.

Admittance: The ratio between current and voltage in r.m.s. terms for sinusoidally varying quantities.

Admittance operator: The ratio between current and voltage in operational terms.

Table 2.1 Electrotechnical symbols and units

Quantity	*Quantity symbol*	*Unit name*	*Unit symbol*
Admittance	Y	siemens	S
Ampere-turn	—	ampere-turn	A-t
Angular frequency	$\omega = 2\pi f$	radian/second	rad/s
Capacitance	C	farad	F
Charge	Q	coulomb	C
Conductance	G	siemens	S
Conductivity	γ, $\sigma\psi$	siemens/metre	S/m
Current	I	ampere	A
Current density, linear	A	ampere/metre	A/m
Current density, surface	J	ampere/metre-square	A/m^2
Electric field strength	E	volt/metre	V/m
Electric flux	Q	coulomb	C
Electric flux density	D	coulomb/metre-square	C/m^2
Electric space constant	ϵ_0	farad/metre	F/m
Electromotive force	E	volt	V
Force	F, f	newton	N
Frequency	f	hertz	Hz
Impedance	Z	ohm	Ω
Inductance, mutual	L_{jk}, M	henry	H
Inductance, self-	L	henry	H
Linkage		weber-turn	Wb-t
Loss angle	$\delta\psi$	radian	rad
Magnetic field strength	H	ampere/metre	A/m
Magnetic flux	Φ	weber	Wb
Magnetic flux density	B	tesla	T
Magnetic space constant	$\mu_0 = 4\pi/10^7$	henry/metre	H/m
Magnetomotive force	F	ampere (-turn)	A, A-t
Period	T	second	s
Permeability, absolute	$\mu = \mu_r\mu_0$	henry/metre	H/m
Permeability, free space	μ_0	henry/metre	H/m
Permeability, relative	μ_r	—	—
Permeance	Λ	weber/ampere (-turn)	Wb/A, Wb/A-t
Permittivity, absolute	$\epsilon = \epsilon_r\epsilon_0$	farad/metre	F/m
Permittivity, free space	ϵ_0	farad/metre	F/m
Permittivity relative	ϵ_r	—	—
Phase angle	$\phi\psi$	radian	rad
Potential	V	volt	V
Potential difference	V, U	volt	V
Potential gradient	E	volt/metre	V/m
Power, active	P	watt	W
Power, apparent	S	volt-ampere	V-A
Power, reactive	Q	var	var
Quantity	Q	coulomb	C
Reactance	X	ohm	Ω
Reluctance	S	ampere (-turn)/weber	A/Wb, A-t/Wb
Resistance	R	ohm	Ω
Resistivity	$\rho\psi$	ohm-metre	Ω-m
Susceptance	B	siemens	S
Time constant	$\tau\psi$	second	s
Voltage	V	volt	V
Voltage gradient	E	volt/metre	V/m

Ampere-turns: The product of the number of turns of a circuit and the current flowing in them.

Angular frequency: The number of periods per second of a periodically varying quantity multiplied by 2π.

Capacitance: The property of a conducting body by virtue of which an electric charge has to be imparted to it to produce a difference of electrical potential between it and the surrounding bodies. The ratio between the charge on a conductor and its potential when all neighbouring conductors are at zero (earth) potential. The ratio between the charge on each electrode of a capacitor and the potential difference between them.

Capacitor: A device having capacitance as a chief property.

Charge: The excess of positive or negative electricity on a body or in space.

Coercive force: The demagnetising force required to reduce to zero the remanent flux density in a magnetic body.

Coercivity: The value of the coercive force when the initial magnetisation has the saturation value.

Complexor: A non-vectorial quantity expressible in terms of a complex number.

Conductance: For steady direct currents, the reciprocal of the resistance. For sinusoidal alternating currents, the resistance divided by the square of the impedance.

Conductivity: The reciprocal of resistivity.

Core loss (iron loss): The loss in a magnetic body subject to changing magnetisation, resulting from hysteresis and eddy current effects.

Current: The flow or transport of electric charges along a path or around a circuit.

Current density: The current per unit area of a conductor, or per unit width of an extended conductor.

Diamagnetic: Having a permittivity less than that of free space.

Dielectric loss: The loss in an insulating body, resulting from hysteresis, conduction and absorption.

Displacement current: The current equivalent of the rate of change of electric flux with time.

Eddy current: The current electromagnetically induced in a conductor lying in a changing magnetic field.

Electric field: The energetic state of the space between two oppositely charged conductors.

Electric field strength: The mechanical force per unit charge on a very small charge placed in an electric field. The negative voltage gradient.

Electric flux: The electric field, equal to the charge, between oppositely charged conductors.

Electric flux density: The electric flux per unit area.

Electric space constant: The permittivity of free space.

Electric strength: The property of an insulating material which enables it to withstand electric stress; or the stress that it can withstand without breakdown.

Electric stress: The electric field intensity, which tends to break down the insulating property of an insulating material.

Electromagnetic field: A travelling field having electric field and magnetic field components and a speed of propagation depending on the electrical properties of the ambient medium.

Electromagnetic induction: The production of an electromotive force in a circuit by a change of magnetic linkage through the circuit. The e.m.f. so produced is an *induced e.m.f.*, and any current that may result therefrom is an *induced current*.

Electromotive force (*e.m.f.*): That quality which tends to cause a movement of charges around a circuit. The direction is that of the movement of positive charges. E.m.f. is measured by the amount of energy developed by transfer of unit positive charge. The term is applied to sources that convert electrical energy to or from some other kind (chemical, mechanical, thermal, etc.).

Ferromagnetic: Having a permeability much greater than that of free space, and varying with the magnetic flux density.

Force: The cause of the mechanical displacement, motion, acceleration and deformation of massive bodies.

Frequency: The number of repetitions of a cyclically time-varying quantity in unit time.

Hysteresis: The phenomenon by which an effect in a body depends not only on the present cause, but also on the previous state of the body. In *magnetisation* a flux density produced by a given magnetic field intensity depends on the previous magnetisation history. A comparable effect occurs in the *electrification* of insulating materials. In cyclic changes hysteresis is the cause of energy loss.

Immitance: A circuit property that can be either impedance or admittance.

Impedance: The ratio between voltage and current in r.m.s. terms for sinusoidally varying quantities.

Impedance operator: The ratio between voltage and current in operational terms.

Inductance: The property of a circuit by virtue of which the passage of a current sets up magnetic linkage and stores magnetic energy. If the linkage of a circuit arises from the current in another circuit, the property is called *mutual inductance*.

Inductor: A device having inductance as a chief property.

Insulation resistance: The resistance under prescribed conditions between two conductors or conducting systems normally separated by an insulating medium.

I^2R loss (copper loss): The loss (converted into heat) due to the passage of a current through the resistance of a conductor.

Line of flux (line of force): A line drawn in a field to represent the direction of the flux at any point.

Linkage: The summation of the products of elements of magnetic flux and the number of turns of the circuit they embrace in a given direction.

Loss angle: The phase angle by which the current in a reactor fails to lead (or lag) the voltage by $\frac{1}{2}\pi$ rad under sinusoidal conditions. The tangent of this angle is called the *loss tangent*.

Magnetic circuit: The closed path followed by a magnetic flux.

Magnetic field: The energetic state of the space surrounding an electric current.

Magnetic field strength: The cause at any point of a magnetic circuit of the magnetic flux density there.

Magnetic flux: The magnetic field, equal to the summation of flux density and area, around a current. A phenomenon in the neighbourhood of currents or magnets. The magnetic flux through any area is the surface integral of the magnetic flux density through the surface. Unit magnetic flux is that flux, the removal of which from a circuit of unit resistance causes unit charge to flow in the circuit; or in an open turn produces a voltage–time integral of unity.

Magnetic flux density: The magnetic flux per unit area at a point in a magnetic field, the area being oriented to give a maximum value to the flux. The normal to the area is the direction of the flux at the point. The direction of the current produced in the electric circuit on removal of the flux, and the positive direction of the flux, have the relation of a right-handed screw.

Magnetic leakage: That part of a magnetic flux which follows such a path as to make it ineffective for the purpose desired.

Magnetic potential difference: A difference between the magnetic states existing at two points which produces a magnetic field between them. It is equal to the line integral of the magnetic field intensity between the points, except in the presence of electric currents.

Magnetic space constant: The permeability of free space.

Magnetising force: The same as magnetic field strength.

Magnetomotive force: Along any path, the line integral of the magnetic field strength along that path. If the path is closed, the line integral is equal to the total magnetising current in ampere-turns.

Paramagnetic: Having a permeability greater than that of free space.

Period: The time taken by one complete cycle of a waveform.

Permeability: The ratio of the magnetic flux density in a medium or material at a point to the magnetic field strength at the point. The *absolute permeability* is the product of the *relative permeability* and the *permeability of free space* (magnetic space constant).

Permeance: The ratio between the magnetic flux in a magnetic circuit and the magnetomotive force. The reciprocal of reluctance.

Permittivity: The ratio between the electric flux density in a medium or material at a point and the electric field strength at the point. The *absolute permittivity* is the product of the *relative permittivity* and the *permittivity of free space* (electric space constant).

Phase angle: The angle between the phasors that represent two alternating quantities of sinusoidal waveform and the same frequency.

Phasor: A sinusoidally varying quantity represented in the form of a complex number.

Polarisation: The change of the electrical state of an insulating material under the influence of an electric field, such that each small element becomes an electric dipole or doublet.

Potential: The electrical state at a point with respect to potential zero (normally taken as that of the earth). It is measured by the work done in transferring unit charge from pontential zero to the point.

Potential difference: A difference between the electrical states existing at two points tending to cause a movement of positive charges from one point to the other. It is measured by the work done in transferring unit charge from one point to the other.

Potential gradient: The potential difference per unit length in the direction in which it is a maximum.

Power: The rate of transfer, storage, conversion or dissipation of energy. In sinusoidal alternating current circuits the *active power* is the mean rate of energy conversion; the *reactive power* is the peak rate of circulation of stored energy; the *apparent power* is the product of r.m.s. values of voltage and current.

Power factor: The ratio between active power and apparent power. In sinusoidal alternating current circuits the power factor is cos ϕ, where ϕ is the phase angle between voltage and current waveforms.

Quantity: The product of the current and the time during which it flows.

Reactance: In sinusoidal alternating current circuits, the quantity ωL or $1/\omega C$, where L is the inductance, C is the capacitance and ω is the angular frequency.

Reactor: A device having reactance as a chief property; it may be an inductor or a capacitor. A *nuclear reactor* is a device in which energy is generated by a process of nuclear fission.

Reluctance: The ratio between the magnetomotive force acting around a magnetic circuit and the resulting magnetic flux. The reciprocal of permeance.

Remanence: The remanent flux density obtained when the initial magnetisation reaches the saturation value for the material.

Remanent flux density: The magnetic flux density remaining in a material when, after initial magnetisation, the magnetising force is reduced to zero.

Residual magnetism: The magnetism remaining in a material after the magnetising force has been removed.

Resistance: That property of a material by virtue of which it resists the flow of charge through it, causing a dissipation of energy as heat. It is equal to the constant potential difference divided by the current produced thereby when the material has no e.m.f. acting within it.

Resistivity: The resistance between opposite faces of a unit cube of a given material.

Resistor: A device having resistance as a chief property.

Susceptance: The reciprocal of reactance.

Time constant: The characteristic time describing the duration of a transient phenomenon.

Voltage: The same as potential difference.

Voltage gradient: The same as potential gradient.

Waveform: The graph of successive instantaneous values of a time-varying physical quantity.

2.2 Thermal effects

2.2.1 Resistance

That property of an electric circuit which determines for a given current the rate at which electrical energy is converted into heat is termed *resistance*. A device whose chief property is resistance is a *resistor*, or, if variable, a *rheostat*. A current I flowing in a resistance R develops heat at the rate

$$P = I^2 R \text{ joule/second or watts}$$

a relation expressing *Joule's law*.

2.2.1.1 Voltage applied to a resistor

In the absence of any energy storage effects (a physically unrealisable condition), the current in a resistor of value R is I when the voltage across it is V, in accordance with the relation $I = V/R$. If a *steady* p.d. V be suddenly applied to a resistor R, the current instantaneously assumes the value given, and energy is expended at the rate $P = I^2 R$ watts, continuously. No transient occurs. If a constant frequency, constant amplitude *sine wave* voltage v is applied, the current i is at every instant given by $i = v/R$, and in consequence the current has also a sine waveform, provided that the resistance is linear. The instantaneous rate of energy dissipation depends on the instantaneous current: it is $p = vi = i^2 R$. Should the applied voltage be non-sinusoidal, the current has (under the restriction mentioned) an exactly similar waveform. The three cases are illustrated in *Figure 2.3*.

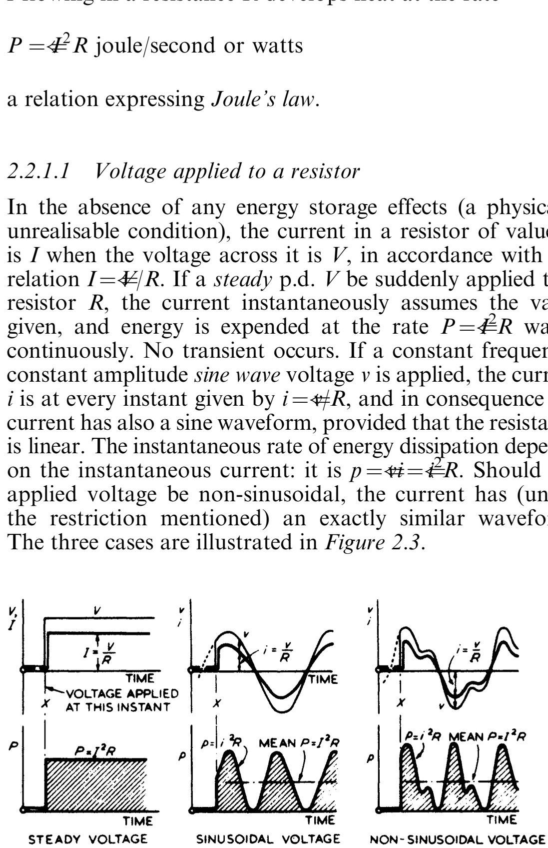


Figure 2.3 Voltage applied to a pure resistor

In the case of alternating waveform, the average rate of energy dissipation is given by $P = I^2R$, where I is the *root-mean-square* current value.

2.2.1.2 *Voltage–current characteristics*

For a given resistor R carrying a constant current I, the p.d. is $V = IR$. The ratio $R = V/I$ may or may not be invariable. In some cases it is sufficient to assume a degree of constancy, and calculation is generally made on this assumption. Where the variations of resistance are too great to make the assumption reasonably valid, it is necessary to resort to less simple analysis or to graphical methods.

A constant resistance is manifested by a constant ratio between the voltage across it and the current through it, and by a straight-line graphical relation between I and V (*Figure 2.4*(*a*)), where $R = V/I = \cot\,\theta$. This case is typical of metallic resistance wires at constant temperature.

Certain circuits exhibit *non-linear* current–voltage relations (*Figure 2.4*(*b*)). The non-linearity may be *symmetrical* or *asymmetrical*, in accordance with whether the conduction characteristics are the same or different for the two current-flow directions. Rectifiers are an important class of non-linear, asymmetrical resistors.

A hypothetical device having the current–voltage characteristic shown in *Figure 2.4*(*c*) has, at an operating condition represented by the point P, a current I_d and a p.d. V_d. The ratio $R_d = V_d/I_d$ is its *d.c. resistance* for the given condition. If a small alternating voltage Δv_a be applied under the same condition (i.e. superimposed on the p.d. V_d), the current will fluctuate by Δi_a and the ratio $r_a = \Delta v_a/\Delta i_a$ is the *a.c.* or *incremental resistance* at P. The d.c. resistance is also obtainable from $R_d = \cot\,\theta$, and the a.c. resistance from $r_a = \cot\,\alpha$. In the region of which Q is a representative point, the a.c. resistance is *negative*, indicating that the device is capable of giving a small output of a.c. power, derived from its greater d.c. input. It remains in sum an energy dissipator, but some of the energy is returnable under suitable conditions of operation.

2.2.1.3 *D.c. or ohmic resistance: linear resistors*

The d.c. or ohmic resistance of linear resistors (a category confined principally to metallic conductors) is a function of the dimensions of the conducting path and of the *resistivity* of the material from which the conductor is made. A wire of length l, cross-section a and resistivity ρ has, at constant given temperature, a resistance

$$R = \rho l/a \text{ ohms}$$

where ρ, l and a are in a consistent system of dimensions (e.g. l in metres, a in square metres, ρ in ohms per 1 m length and 1 m^2 cross-section—generally contracted to ohm-metres). The expression above, though widely applicable, is true only on the assumption that the current is uniformly distributed over the cross-section of the conductor and flows in paths parallel to the boundary walls. If this assumption is inadmissible, it is necessary to resort to integration or the use of current-flow lines. *Figure 2.5* summarises the expressions for the resistance of certain arrangements and shapes of conductors.

Resistivity The resistivity of conductors depends on their composition, physical condition (e.g. dampness in the case of non-metals), alloying, manufacturing and heat treatment, chemical purity, mechanical working and ageing. The *resistance-temperature coefficient* describes the rate of change of resistivity with temperature. It is practically 0.004 Ω/°C at 20°C for copper. Most pure metals have a resistivity that rises with temperature. Some alloys have a very small coefficient. Carbon is notable in that its resistivity decreases markedly with temperature rise, while uranium dioxide has a resistivity which falls in the ratio 50:1 over a range of a few hundred degrees. *Table 2.2* lists the resistivity ρ and the resistance-temperature coefficient α for a number of representative materials. The effect of temperature is assessed in accordance with the expressions

$$R_1 = R_0(1 + \alpha\theta_1); \quad R_2/R_1 = (1 + \alpha\theta_2)/(1 + \alpha\theta_1);$$

or

$$R_2 = R_1[1 + \alpha(\theta_2 - \theta_1)]$$

where R_0, R_1 and R_2 are the resistances at temperatures 0, θ_1 and θ_2, and α is the resistance-temperature coefficient at 0°C.

2.2.1.4 *Liquid conductors*

The variations of resistance of a given aqueous solution of an electrolyte with temperature follow the approximate rule:

$$R_\theta = R_0/(1 + 0.03\theta)$$

where θ is the temperature in degrees Celsius. The conductivity (or reciprocal of resistivity) varies widely with the percentage strength of the solution. For low concentrations the variation is that given in *Table 2.2*.

2.2.1.5 *Frequency effects*

The resistance of a given conductor is affected by the frequency of the current carried by it. The simplest example is that of an isolated wire of circular cross-section. The inductance of the central parts of the conductor is greater than that of the outside skin because of the additional flux linkages due to the internal magnetic flux lines. The impedance of the central parts is consequently greater, and the current flows mainly at and near the surface of the conductor, where the impedance is least. The useful cross-section of the conductor is less than the actual area, and the effective resistance is consequently higher. This is called the *skin effect*. An analogous phenomenon, the *proximity effect*, is due to mutual inductance between conductors arranged closely parallel to one another.

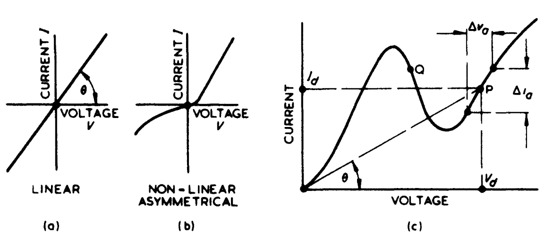


Figure 2.4 Current–voltage characteristics

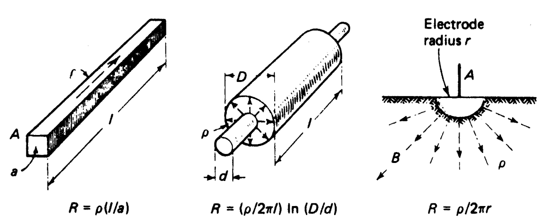


Figure 2.5 Resistance in particular cases

Table 2.2 Conductivity of aqueous solutions* (mS/cm)

Concentration (%)	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h, j*	*k*	*l, m*
1	40	18	12	10	10	8	6	4	3	3
2	72	35	23	20	20	16	12	8	6	6
3	102	51	34	30	30	24	18	12	9	8
4	130	65	44	39	39	32	23	16	11	10
5		79	55	48	47	39	28	20	13	11
7.5		110	79	69	67	54	39	29	18	16
10			99	90	85	69		31	22	20

*(a) NaOH, caustic soda; (b) NH_4Cl, sal ammoniac; (c) NaCl, common salt; (d) $NaNO_3$, Chilean saltpetre; (e) $CaCl_2$, calcium chloride; (f) $ZnCl_2$, zinc chloride; (g) $NaHCO_3$, baking soda; (h) Na_2CO_3, soda ash; (j) Na_2SO_4, Glauber's salt; (k) $Al_2(SO_4)_3K_2SO_4$, alum; (l) $CuSO_4$, blue vitriol; (m) $ZnSO_4$, white vitriol.

The effects depend on conductor size, frequency f of the current, resistivity ρ and permeability μ of the material. For a circular conductor of diameter d the increase of effective resistance is proportional to $d^2f\mu/\rho$. At power frequencies and for small conductors the effect is negligible. It may, however, be necessary to investigate the skin and proximity effects in the case of large conductors such as bus-bars.

2.2.1.6 Non-linear resistors

Prominent among non-linear resistors are electric arcs; also silicon carbide and similar materials.

Arcs An electric arc constitutes a conductor of somewhat vague dimensions utilising electronic and ionic conduction in a gas. It is strongly affected by physical conditions of temperature, gas pressure and cooling. In air at normal pressure a d.c. arc between copper electrodes has the voltage–current relation given approximately by $V = 30 + 10/I + l[1 + 3/I]10^3$ for a current I in an arc length l metres. The expression is roughly equivalent to 10 V/cm for large currents and high voltages. The current density varies between 1 and 1000 A/mm^2, being greater for large currents because of the *pinch* effect. See also Section 2.7.

Silicon carbide Conducting pieces of this material have a current–voltage relation expressed approximately by $I = KV^x$, where x is usually between 3 and 5. For rising voltage the current increases very rapidly, making silicon carbide devices suitable for circuit protection and the discharge of excess transmission-line surge energy.

2.2.2 Heating and cooling

The *heating* of any body such as a resistor or a conducting circuit having inherent resistance is a function of the losses within it that are developed as heat. (This includes core and dielectric as well as ohmic I^2R losses, but the effective value of R may be extended to cover such additional losses.) The *cooling* is a function of the facilities for heat dissipation to outside media such as air, oil or solids, by radiation, conduction and convection.

2.2.2.1 Rapid heating

If the time of heating is short, the cooling may be ignored, the temperature reached being dependent only on the rate of development of heat and the thermal capacity. If p is the heat development per second in joules (i.e. the power in watts), G the mass of the heated body in kilograms and c its specific heat in joules per kilogram per kelvin, then

$$Gc \cdot d\theta = p \cdot dt, \quad \text{giving} \quad \theta = (1/Gc)\int p \cdot dt$$

For steady heating, the temperature rise is p/Gc in Kelvin per second.

Standard annealed copper is frequently used for the windings and connections of electrical equipment. Its density is $G = 8900$ kg/m^3 and its resistivity at 20°C is 0.017 μΩ-m; at 75°C it is 0.021 μΩ-m. A conductor worked at a current density J (in amperes per square metre) has a specific loss (watts per kilogram) of $\rho J^2/8900$. If $J = 2.75$ MA/m^2 (or 2.75 A/mm^2), the specific loss at 75°C is 17.8 W/kg, and its rate of self-heating is 17.8/375 = 0.048°C/s.

2.2.2.2 Continuous heating

Under prolonged steady heating a body will reach a temperature rise above the ambient medium of $\theta_m = p/A\lambda$, where A is the cooling surface area and λ the specific heat dissipation (joules per second per square metre of surface per degree Celsius temperature rise above ambient). The expression is based on the assumption, roughly true for moderate temperature rises, that the rate of heat emission is proportional to the temperature rise. The specific heat dissipation λ is compounded of the effects of *radiation*, *conduction* and *convection*.

Radiation The heat radiated by a surface depends on the absolute temperature T (given by $T = \theta + 273$, where θ is the Celsius temperature), and on its character (surface smoothness or roughness, colour, etc.). The Stefan law of heat radiation is

$$p_r = 5.7eT^4 \times 10^{-8} \text{ watts per square metre}$$

where e is the coefficient of radiant emission, always less than unity, except for the perfect 'black body' surface, for which $e = 1$. The radiation from a body is independent of the temperature of the medium in which it is situated. The process of radiation of a body to an exterior surface is accompanied by a re-absorption of part of the energy when re-radiated by that surface. For a small spherical radiating body inside a large and/or black spherical cavity, the radiated power is given by the Stefan-Boltzmann law:

$$p_r = 5.7e_1[T_1^4 - T_2^4]10^{-8} \text{ watts per square metre}$$

where T_1 and e_1 refer to the body and T_2 to the cavity.

The emission of radiant heat from a perfect black body surface is independent of the roughness or corrugation of the surface. If $e < 1$, however, there is some increase of radiation if the surface is rough.

Conduction The conduction of heat is a function of the thermal or temperature gradient and the thermal resistivity, the latter being defined as the temperature difference in degrees Celsius across a path of unit length and unit section required for the continuous transmission of 1 W. Thus, the heat conducted per unit area along a path of length x in a material of thermal resistivity ρ for a temperature difference of θ is

$$p_{\mathrm{d}} = \theta/\rho x \text{ watts}$$

Resistivities for metals are very low. For insulating materials such as paper, $\rho = 5$–10; for *still* air, $\rho = 20$ W per °C per m and per m^2, approximately.

Convection Convection currents in liquids and gases (e.g. oil and air) are always produced near a heated surface unless baffled. Convection adds greatly to heat dissipation, especially if artificially stimulated (as in force cooling by fans). Experiment shows that a rough surface dissipates heat by convection more readily than a smooth one, and that high fluid speeds are essential to obtain *turbulence* as opposed to *stream-line* flow, the former being much more efficacious.

Convection is physically a very complex phenomenon, as it depends on small changes in buoyancy resulting from temperature rise due to heating. Formulae for dissipation of heat by convection have a strongly empirical basis, the form and orientation of the convection surfaces having considerable influence.

Cooling coefficient For electrical purposes the empirically derived coefficient of emission λ, or its reciprocal $1/\lambda$, are employed for calculations on cooling and temperature rise of wires, resistors, machines and similar plant.

2.2.2.3 Measurement of temperature rise

The temperature rise of a device developing heat can be measured (a) by a thermometer placed in contact with the surface whose temperature is required, (b) by resistance-temperature detectors or thermocouples on the surface of, or embedded in, the device, or (c) by the measurement of resistance (in the case of conducting circuits), using the known resistance-temperature coefficient. These methods measure different temperatures, and do not give merely alternative estimates of the same thing.

2.2.2.4 Heating and cooling cycles

In some cases a device (such as a machine or one of its parts) developing internal heat may be considered as sufficiently homogeneous to apply the exponential law. Suppose the device to have a temperature rise θ after the lapse of a time t. In an element of time $\mathrm{d}t$ a small temperature rise $\mathrm{d}\theta$ takes place. The heat developed is $p{\cdot}\mathrm{d}t$, the heat stored is $Gh{\cdot}\mathrm{d}\theta$, and the heat dissipated is $A\theta\lambda{\cdot}\mathrm{d}t$. Since the heat stored and dissipated together equal the total heat produced,

$$Gh \cdot \mathrm{d}\theta + A\theta\lambda \cdot \mathrm{d}t = p \cdot \mathrm{d}t$$

the solution of which is

$$\theta = \theta_{\mathrm{m}}[1 - \exp(-t/\tau)]$$

where θ_{m} is the final steady temperature rise, calculated from $\theta_{\mathrm{m}} = p/A\lambda$, and $\tau = Gh/A\lambda$ is called the *heating time constant*. For the lapse of time t equal to the time constant

$$\theta = \theta_{\mathrm{m}}[1 - \exp(-1)] = 0.632\theta_{\mathrm{m}}$$

When a heated body cools owing to a reduction or cessation of internal heat production, the temperature–time relation is the exponential function

$$\theta = \theta_{\mathrm{m}} \exp(-t/\tau_1)$$

where τ_1 is the *cooling time constant*, not necessarily the same as that for heating conditions.

Both heating and cooling as described are examples of *thermal transients*, and the laws governing them are closely analogous to those concerned with transient electric currents, in which exponential time relations also occur.

2.2.2.5 Fusing currents

For a given diameter d, the heat developed by a wire carrying a current I is inversely proportional to d^3, because an increase of diameter reduces the current density in proportion to the increase of area, and the emitting surface is increased in proportion to the diameter. The temperature rise is consequently proportional to I^2/d^3. If the temperature is raised to the fusing or melting point, $\theta = aI^2/d^3$ and the fusing current is

$$I = \sqrt{(\theta d^3/a)} = kd^{3/2}$$

This is *Preece's law*, from which an estimate may be made of the fusing current of a wire of given diameter, provided that k is known. The exponent 3/2 and the value of k are both much affected by enclosure, conduction of heat by terminals, and similar physical conditions.

It is obvious that any rule regarding suitable current densities giving a value regardless of the diameter is likely to be uneconomically low for small wires and excessive for large ones. Further, the effects of length and enclosure make a direct application of Preece's law unreliable. For small wires the exponent x in the term d^x may be 1.25–1.5, and for larger wires it may exceed 1.5.

2.2.2.6 Thermo-e.m.f.s

An effect known as the *thermoelectric effect* or *Seebeck effect* is that by which an e.m.f. is developed due to a difference of temperature between two junctions of dissimilar conductors in the same circuit. The *Thomson effect* or *Kelvin effect* is (a) that an e.m.f. is developed due to a difference of temperature between two parts of the same conductor, and (b) that an absorption or liberation of heat takes place when a current flows from a hotter to a colder part of the same material. The *Peltier effect* describes the liberation or absorption of heat at a joint where current passes from one material to another, whereby the joint becomes heated or cooled.

In *Figure 2.6*(*a*)–(*c*) the symbols are absolute temperature T, thermo-e.m.f. E and rate of heat production or absorption Q. The *Seebeck coefficient* (a) is the e.m.f. per degree difference between hot and cold junctions:

$$\alpha_{\mathrm{S}} = \Delta E/\Delta T$$

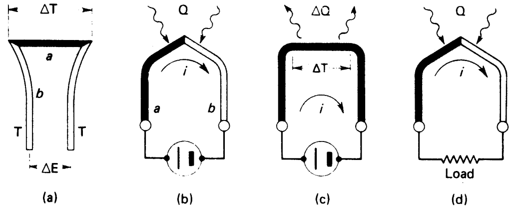


Figure 2.6 Thermo-e.m.f.

Typical e.m.f.s for a number of common junctions are given in *Table 2.3*. In the Peltier effect (*b*) a rate of heat generation (reversible, and distinct from the irreversible I^2R heat) results from the passage of a current *i* through the different conductors A and B. The *Peltier coefficient* is

$$\alpha_P = Q/i$$

The Thomson effect (*c*) concerns the rate of reversible heat when a current *i* flows through a length of homogeneous conductor across which there is a temperature difference. The *Thomson coefficient* is

$$\alpha_T = \Delta Q/i \cdot \Delta T$$

The relation between the Seebeck and Peltier coefficients is important: it is

$$\alpha_S = \alpha_P/T$$

The Seebeck coefficient is the more easily measured, but the Peltier coefficient determines the cooling effect of a thermoelectric refrigerator.

Table 2.3 Thermocouple e.m.f.s (mV): cold junction at 0 °C

Hot-junction temperature (°C)	*Platinum/* $^{87}Pt-{}^{13}Rh$	*Chromel/ Alumel*	*Iron/ Eureka*	*Copper/ Eureka*
100	0.65	4.1	5	4
200	1.46	8.1	11	9
400	3.40	16.4	22	21

2.2.2.7 *Thermoelectric devices*

If a current flows through a thermocouple (*Figure 2.6*(*b*)), with one junction in thermal contact with a heat sink, the other removes heat from a source. The couple must comprise conductors with positive and negative Seebeck coefficients, respectively. The arrangement is a *refrigerator* with the practical advantages of simplicity and silence.

A heat source applied to a junction develops an e.m.f. that will circulate a current in an external load (*Figure 2.6*(*d*)). If semiconductors of low thermal conductivity are used in place of metals for the couple elements, a better efficiency is obtainable because heat loss by conduction is reduced. The couples in *Figure 2.7*(*a*) of a thermoelectric power generator are constructed with p- and n-type materials. The efficiency, limited by Carnot cycle considerations, does not at present exceed 10 %.

A thermocouple generator in which one element is an electron stream or plasma is the *thermionic generator*, in effect a diode with flat cathode and anode very close together. By virtue of their kinetic energy, electrons emitted from the cathode reach the anode against a small, negative anode potential, providing current for an external circuit (*Figure 2.7*(*b*)). The work function of the anode material must be less than for the cathode. The device is a heat engine operating over the cathode–anode temperature fall, with electrons providing the 'working fluid'.

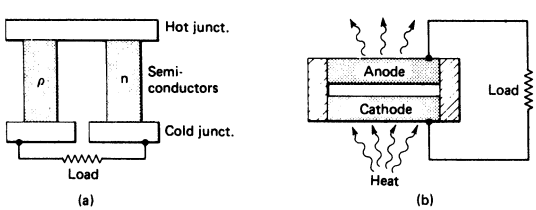


Figure 2.7 Thermoelectric devices

Outputs of 2 kW/m^2 at an efficiency of 25 % may be reached when the device has been fully developed and the space charge effects overcome. Cathode heating by solar energy is a possibility.

2.3 Electrochemical effects

2.3.1 Electrolysis

If a liquid conductor undergoes chemical changes when a current is passed through it, the effect is ascribed to the movement of constituent parts of the molecules of the liquid *electrolyte*, called *ions*, which have a positive or negative electric charge. Positive ions move towards the negative electrode (*cathode*) and negative ions to the positive electrode (*anode*). The ionic movement is the reason for the current conduction. Ions reaching the electrodes have their charges neutralised and may be subject to chemical change. Hydrogen and metal ions are electropositive: non-metals of the chlorine family (Cl, Br, I and F) and acid radicals (such as SO_4 and NO_3) form negative ions in solution. As examples, hydrochloric acid, HCl, forms H^+ and Cl^- ions; sulphuric acid forms $2H^+$ and SO_4^{--} ions; and sodium hydroxide, NaOH, yields Na^+ and OH^- ions. The products of electrolysis depend on the nature of the electrolyte. Basic solutions of sodium or similar hydroxides produce H_2 and O_2 gases at the cathode and anode, respectively. Acid solutions give products depending on the nature of the electrodes. Solutions of metal salts with appropriate electrodes result in electrodeposition.

The mass of the ion of an element of radical deposited on, dissolved from or set free at either electrode is proportional to the quantity of electricity passed through the electrolytic cell and to the ionic weight of the material, and inversely proportional to the valency of the ion; whence the mass m_e in kilogram-equivalents is the product (*z* in kilogram-equivalents per coulomb) × (*Q* in coulombs). The value of *z* is a natural constant 0.001 036. Representative figures (for convenience in milligrams per coulomb) are given in *Table 2.4*.

To pass a current through an electrolyte, a p.d. must be applied to the electrodes to overcome the drop in resistance of the electrolyte, and to overcome the e.m.f. of *polarisation*. The latter is due to a drop across a thin film of gas, or through a strong ionic concentration, at an electrode.

Every chemical reaction may be represented as two electrode reactions. The algebraic p.d. between the two is a measure of the reactivity. A highly negative p.d. represents a spontaneous reaction that might be utilised to generate a current. A high positive p.d. represents a reaction requiring an external applied p.d. to maintain it.

2.3.1.1 *Uses of electrolysis*

Ores of copper, zinc and cadmium may be electrolytically treated with sulphuric acid to deposit the metal. Copper may be deposited by use of a low voltage at the cathode, while oxygen is emitted at the anode. Electrorefining by deposition may be employed with copper, nickel, tin, silver, etc., produced by smelting or electrowinning, by using the impure metal as anode, which is dissolved away and redeposited on the cathode, leaving at the bottom of the cell the impurities in the form of sludge. Electroplating is similar to electrorefining except that pure metal or alloy is used as the anode.

Table 2.4 Electrochemical equivalents z (mg/C)

Element	*Valency*	*z*	*Element*	*Valency*	*z*
H	1	0.010 45	Zn	2	0.338 76
Li	1	0.071 92	As	3	0.258 76
Be	2	0.046 74	Se	4	0.204 56
O	2	0.082 90	Br	1	0.828 15
F	1	0.196 89	Sr	2	0.454 04
Na	1	0.238 31	Pd	4	0.276 42
Mg	2	0.126 01	Ag	1	1.117 93
Al	3	0.093 16	Cd	2	0.582 44
Si	4	0.072 69	Sn	2	0.615 03
S	2	0.166 11	Sn	4	0.307 51
S	4	0.083 06	Sb	3	0.420 59
S	6	0.055 37	Te	4	0.330 60
Cl	1	0.367 43	I	1	1.315 23
K	1	0.405 14	Cs	1	1.377 31
Ca	2	0.207 67	Ba	2	0.711 71
Ti	4	0.124 09	Ce	3	0.484 04
V	5	0.105 60	Ta	5	0.374 88
Cr	3	0.179 65	W	6	0.317 65
Cr	6	0.089 83	Pt	4	0.505 78
Mn	2	0.284 61	Au	1	2.043 52
Fe	1	0.578 65	Au	3	0.681 17
Fe	2	0.289 33	Hg	1	2.078 86
Fe	3	0.192 88	Hg	2	1.039 42
Co	2	0.305 39	Tl	1	2.118 03
Ni	2	0.304 09	Pb	2	1.073 63
Cu	1	0.658 76	Bi	3	0.721 93
Cu	2	0.329 38	Th	4	0.601 35

2.3.2 Cells

2.3.2.1 Primary cells

An elementary cell comprising electrodes of copper (positive) and zinc (negative) in sulphuric acid develops a p.d. between copper and zinc. If a circuit is completed between the electrodes, a current will flow, which acts in the electrolyte to decompose the acid, and causes a production of hydrogen gas round the copper, setting up an e.m.f. of polarisation in opposition to the original cell e.m.f. The latter therefore falls considerably. In practical primary cells the effect is avoided by the use of a *depolariser*. The most widely used primary cell is the Leclanché. It comprises a zinc and a carbon electrode in a solution of ammonium chloride, NH_4Cl. When current flows, zinc chloride, ZnCl, is formed, releasing electrical energy. The NH_4 positive ions travel to the carbon electrode (positive), which is packed in a mixture of manganese dioxide and carbon as depolariser. The NH_4 ions are split up into NH_3 (ammonia gas) and H, which is oxidised by the MnO_2 to become water. The removal of the hydrogen prevents polarisation, provided that the current taken from the cell is small and intermittent.

The *wet* form of Leclanché cell is not portable. The *dry* cell has a paste electrolyte and is suitable for continuous moderate discharge rates. It is exhausted by use or by ageing and drying up of the paste. The 'shelf life' is limited. The *inert* cell is very similar in construction to the dry cell, but is assembled in the dry state, and is activated when required by moistening the active materials. In each case the cell e.m.f. is about 1.5 V.

2.3.2.2 Standard cell

The Weston normal cell has a positive element of mercury, a negative element of cadmium, and an electrolyte of cadmium sulphate with mercurous sulphate as depolariser. The open-circuit e.m.f. at 20°C is about 1.018 30 V, and the e.m.f./temperature coefficient is of the order of – 0.04 mV/°C.

2.3.2.3 Secondary cells

In the *lead–acid* storage cell or accumulator, lead peroxide reacts with sulphuric acid to produce a positive charge at the anode. At the cathode metallic lead reacts with the acid to produce a negative charge. The lead at both electrodes combines with the sulphate ions to produce the poorly soluble lead sulphate. The action is described as

$$\text{Charged}\quad \underset{\text{Brown}}{\overset{+\Leftarrow}{PbO_2}} + \underset{\text{Strong acid}}{2H_2SO_4} + \underset{\text{Grey}}{\overset{-\Leftarrow}{Pb}} \quad \text{Discharged}$$

$$= \underset{\text{Sulphurate}}{\overset{+\Leftarrow}{PbSO_4}} + \underset{\text{Weak acid}}{2H_2O} + \underset{\text{Sulphate}}{\overset{-\Leftarrow}{PbSO_4}}$$

Both electrode reactions are reversible, so that the initial conditions may be restored by means of a 'charging current'.

In the *alkaline* cell, nickel hydrate replaces lead peroxide at the anode, and either iron or cadmium replaces lead at the cathode. The electrolyte is potassium hydroxide. The reactions are complex, but the following gives a general indication:

Charged Discharged

$$2Ni(\overset{+\Leftarrow}{OH})_3 + KOH + \overset{-\Leftarrow}{Fe} = 2Ni(\overset{+\Leftarrow}{OH})_2 + KOH + Fe(\overset{-\Leftarrow}{OH})_2$$

$$\text{or } 2Ni(OH)_3 + KOH + Cd = 2Ni(OH)_3 + KOH + Cd(OH)_2$$

2.3.2.4 Fuel cell

Whereas a storage battery cell contains all the substances in the electrochemical oxidation–reduction reactions involved and has, therefore, a limited capacity, a fuel cell is supplied with its reactants externally and operates continuously as long as it is supplied with fuel. A practical fuel cell for direct conversion into electrical energy is the hydrogen–oxygen cell (*Figure 2.8*). Microporous electrodes serve to bring the gases into intimate contact with the electrolyte (potassium hydroxide) and to provide the cell terminals. The hydrogen and oxygen reactants are fed continuously into the cell from externally, and electrical energy is available on demand.

At the fuel (H_2) electrode, H_2 molecules split into hydrogen atoms in the presence of a catalyst, and these combine with OH^{-} ions from the electrolyte, forming H_2O and releasing electrons *e*. At the oxygen electrode, the oxygen molecules (O_2) combine (also in the presence of a catalyst) with water molecules from the electrolyte and with pairs of electrons arriving at the electrode through the external load from the fuel electrode. Perhydroxyl ions (O_2H^-) and hydroxyl ions (OH^-) are produced: the latter enter the electrolyte, while the more resistant O_2H^- ions, with special catalysts, can be

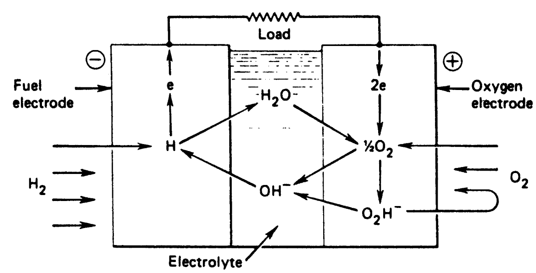


Figure 2.8 Fuel cell

reduced to OH^- ions and oxygen. The overall process can be summarised as:

Fuel electrode	$H_2 + 2OH^- = 2OH_2O + 2e$
Oxygen electrode	$\frac{1}{2}O_2 + H_2O + 2e = 2OH^{-\Leftarrow}$
Net reaction	$H_2 + \frac{1}{2}O_2 \rightarrow 2e \text{ flow} \rightarrow H_2O$

In a complete reaction 2 kg hydrogen and 16 kg oxygen combine chemically (not explosively) to form 18 kg of water with the release of 400 MJ of electrical energy. For each kiloamp/hour the cell produces 0.33 1 of water, which must not be allowed unduly to weaken the electrolyte. The open-circuit e.m.f. is 1.1 V, while the terminal voltage is about 0.9 V, with a delivery of 1 kA/m^2 of plate area.

2.4 Magnetic field effects

The space surrounding permanent magnets and electric circuits carrying currents attains a peculiar state in which a number of phenomena occur. The state is described by saying that the space is threaded by a *magnetic field of flux*. The field is mapped by an arrangement of *lines of induction* giving the strength and direction of the flux. *Figure 2.9* gives a rough indication of the flux pattern for three simple cases of magnetic field due to a current. The diagrams show the conventions of polarity, direction of flux and direction of current adopted. Magnetic lines of induction form closed loops in a *magnetic circuit* linked by the circuit current wholly or in part.

2.4.1 Magnetic circuit

By analogy with the electric circuit, the magnetic flux produced by a given current in a magnetic circuit is found from the magnetomotive force (m.m.f.) and the circuit reluctance. The m.m.f. produced by a coil of N turns carrying a current I is $F = NI$ ampere-turns. This is expended over any closed path linking the current I. At a given point in a magnetic field in free space the m.m.f. per unit length or magnetising force H gives rise to a magnetic flux density $B_0 = \mu_0 H$, where $\mu_0 = 4\pi/10^7$. If the medium in which the field exists has a relative permeability μ_r, the flux density established is

$$B = \mu_r B_0 = \mu_r \mu_0 H = \mu H$$

The summation of $H{\cdot}dl$ round any path linking an N-turn circuit carrying current I is the total m.m.f. F. If the distribution of H is known, the magnetic flux density B or B_0 can be

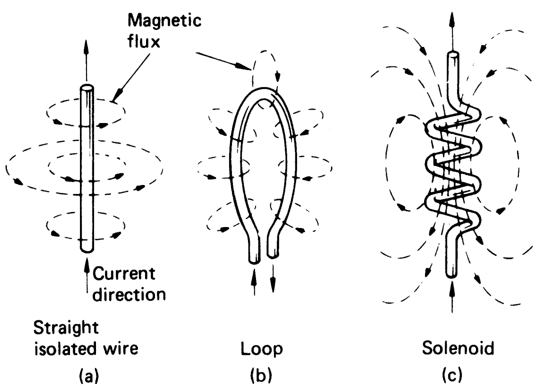


Figure 2.9 Magnetic fields

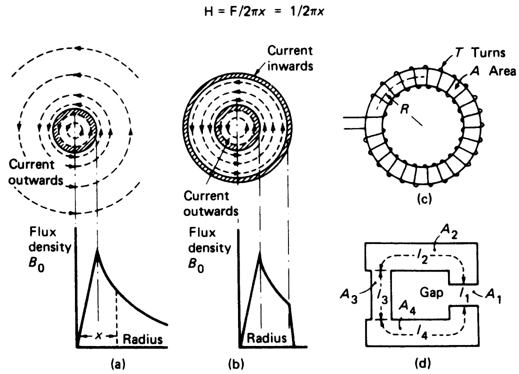


Figure 2.10 Magnetic circuits

found for all points in the field, and a knowledge of the area a of the magnetic path gives $\Phi = Ba$, the total magnetic flux.

Only in a few cases of great geometrical simplicity can the flux due to a given system of currents be found precisely. Among these are the following.

Long straight isolated wire (*Figure 2.10*(*a*)): This is not strictly a realisable case, but the results are useful. Assume a current of 1 A. The m.m.f. around any closed linking path is therefore 1 A-t. Experiment shows that the magnetic field is symmetrical about, and concentric with, the axis of the wire. Around a closed path of radius x metres there will be a uniform distribution of m.m.f. so that

$$H = F/2\pi x = 1/2\pi x (\text{A-t/m}) \Leftarrow$$

Consequently, in free space the flux density (T) at radius x is

$$B_0 = \mu_0 H = \mu_0 / 2\pi x$$

In a medium of constant permeability $\mu = \mu_r \mu_0$ the flux density is $B = \mu_r B_0$. There will be magnetic flux following closed circular paths within the cross-section of the wire itself: at any radius x the m.m.f. is $F = (x/r)^2$ because the circular path links only that part of the (uniformly distributed) current within the path. The magnetising force is $H = F/2\pi x = x/2\pi r^2$ and the corresponding flux density in a non-magnetic conductor is

$$B_0 = \mu_0 H = \mu_0 x / 2\pi r^2$$

and μ_r times as much if the conductor material has a relative permeability μ_r. The expressions above are for a conductor current of 1 A.

Concentric conductors (*Figure 2.10*(*b*)): Here only the inner conductor contributes the magnetic flux in the space between the conductors and in itself, because all such flux can link only the inner current. The flux distribution is found exactly as in the previous case, but can now be summed in defined limits. If the outer conductor is sufficiently thin radially, the flux in the interconductor space, per metre axial length of the system, is

$$\Phi = \Leftarrow \int_r^R \frac{\mu_0}{2\pi x}\, dx = \Leftarrow \frac{\mu_0}{2\pi \psi} \ln \frac{R}{r}$$

Toroid (*Figure 2.10*(*c*)): This represents the closest approach to a perfectly symmetrical magnetic circuit, in which the m.m.f. is distributed evenly round the magnetic path and the m.m.f. per metre H corresponds at all points exactly to the flux density existing at those points. The magnetic flux is therefore wholly confined to the path. Let the mean radius of the toroid be R and its cross-sectional area be A. Then, with N uniformly distributed turns carrying a current I and a toroid core of permeability μ,

$$F = NI; \quad H = F/2\pi R; \quad B = \mu H; \quad \Phi = \mu FA/2\pi R$$

This applies approximately to a long solenoid of length l, replacing R by $1/2\pi$. The permeability will usually be μ_0.

Composite magnetic circuit containing iron (*Figure 2.10*(*d*)): For simplicity practical composite magnetic circuits are arbitrarily divided into parts along which the flux density is deemed constant. For *each* part

$$F = Hl = Bl/\mu = BlA/\mu A = \Phi S$$

where $S = l/\mu A$ is the *reluctance*. Its reciprocal $\Lambda = 1/S = \mu A/l$ is the *permeance*. The expression $F = \Phi S$ resembles $E = IR$ for a simple d.c. circuit and is therefore sometimes called the *magnetic Ohm's law*.

The total excitation for the magnetic circuit is

$$F = H_1 l_1 + H_2 l_2 + H_3 l_3 + \cdots$$

for a series of parts of length $l_1, l_2 \ldots$, along which magnetic field intensities of $H_1, H_2 \ldots$ (A-t/m) are necessary. For free space, air and non-magnetic materials, $\mu_r = 1$ and $B_0 = \mu_0 H$, so that $H = B_0/\mu_0 \simeq 800\,000\, B_0$. This means that an excitation $F = 800\,000$ A-t is required to establish unit magnetic flux density (1 T) over a length $l = 1$ m. For ferromagnetic materials it is usual to employ *B–H* graphs (*magnetisation curves*) for the determination of the excitation required, because such materials exhibit a *saturation* phenomenon. Typical *B–H* curves are given in *Figures 2.11* and *2.12*.

2.4.1.1 Permeability

Certain *diamagnetic* materials have a relative permeability slightly less than that of vacuum. Thus, bismuth has $\mu_r = 0.9999$. Other materials have μ_r slightly greater than unity: these are called *paramagnetic*. Iron, nickel, cobalt, steels, Heusler alloy (61% Cu, 27% Mn, 13% Al) and a number of others of great metallurgical interest have *ferromagnetic* properties, in which the flux density is not directly proportional to the magnetising force but which under suitable conditions are strongly magnetic. The more usual constructional materials employed in the magnetic circuits of electrical machinery may have peak permeabilities in the neighbourhood of 5000–10 000. A group of nickel–iron alloys, including *mumetal* (73% Ni, 22% Fe, 5% Cu), *permalloy* '*C*' (77.4% Ni, 13.3% Fe, 3.7% Mo, 5% Cu) and *hypernik* (50% Ni, 50% Fe), show much higher permeabilities at low densities (*Figure 2.12*). Permeabilities depend on exact chemical composition, heat treatment, mechanical stress and temperature conditions, as well as on the flux density. Values of μ_r exceeding 5×10^5 can be achieved.

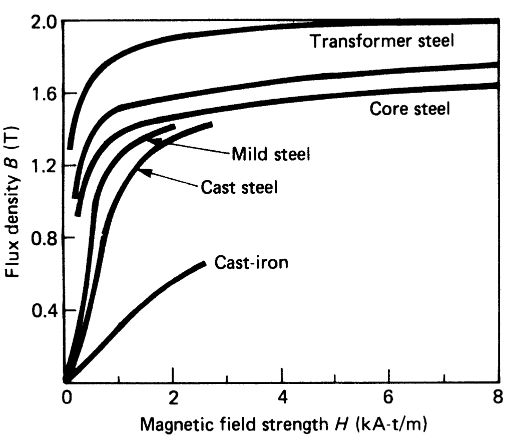


Figure 2.11 Magnetisation curves

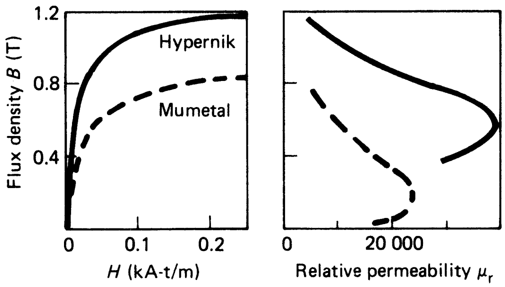


Figure 2.12 Magnetisation and permeability curves

2.4.1.2 Core losses

A ferromagnetic core subjected to cycles of magnetisation, whether alternating (reversing), rotating or pulsating, exhibits *hysteresis*. *Figure 2.13* shows the cycle *B–H* relation typical of this phenomenon. The significant quantities *remanent flux* density and *coercive force* are also shown. The area of the *hysteresis loop* figure is a measure of the energy loss in the cycle per unit volume of material. An empirical expression for the *hysteresis loss* in a core taken through f cycles of magnetisation per second is

$$p_h = k_h f B_m^x \text{ watts per unit mass or volume}$$

Here B_m is the maximum induction reached and k_h is the hysteretic constant depending on the molecular quality and structure of the core metal. The exponent x may lie between 1.5 and 2.3. It is often taken as 2.

A further cause of loss in the same circumstances is the *eddy current loss*, due to the I^2R losses of induced currents. It can be shown to be

$$p_e = k_e t^2 F^2 B^2 \text{ watts per unit mass or volume}$$

the constant k_e depending on the resistivity of the metal and t being its thickness, the material being laminated to decrease the induced e.m.f. per lamina and to increase the resistance of the path in which the eddy currents flow. In practice, curves of loss per kilogram or per cubic metre for various flux densities are employed, the curves being constructed from the results of

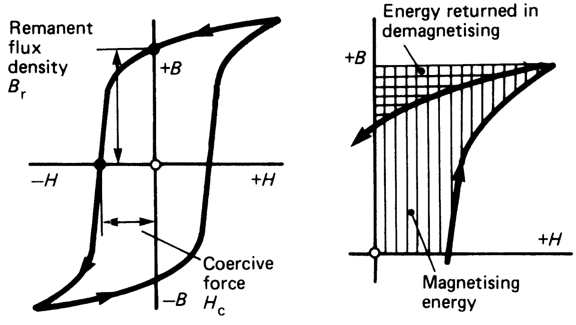


Figure 2.13 Hysteresis

careful tests. It should be noted that hysteresis loss is dependent on the maximum flux density B_m, while the eddy current loss is a function of r.m.s. induced current and e.m.f., and therefore of the r.m.s. flux density B, and not the maximum density B_m.

2.4.1.3 Permanent magnets

Permanent magnets are made from heat-treated alloys, or from ferrites and rare earths, to give the material a large hysteresis loop. *Figure 2.14* shows the demagnetisation B/H quadrant of the loop of a typical material. In use, a magnet produces magnetic energy in the remainder of the magnetic circuit derived from a measure of self-demagnetisation: consequently, the working point of the magnet is on the loop between the coercive force/zero flux point and the zero force/remanent flux point. Different parts of the magnet will work at different points on the loop, owing to leakage, and the conditions become much more complex if the reluctance of the external magnetic circuit fluctuates.

In designing a magnet it is necessary to allow for leakage by use of an m.m.f. allowance F_a (normally not more than 1.25) and a flux allowance Φ_a, which may be anything from 2 to 20, being greater for a high ratio between gap length and gap section.

If H_g is the field strength in gap, l_g the gap length, A_g the gap section, B_m the working density in the magnet, H_m the working demagnetising field strength in the magnet, l_m the magnet length and A_m the magnet section, then it may be shown that

$$H_g = \sqrt{[(B_m H_m / F_a \Phi_a)(A_m l_m / A_g l_g)]}$$

i.e. it is greatest when $B_m H_m$ is a maximum. This occurs for a working point at $(BH)_{max}$. The magnet length and section must be proportioned to suit the alloy and the gap dimensions to secure the required condition. The section is $A_m = (H_g A_g \Phi_a / B_m)$ and the length $l_m = (H_g l_g F_a / H_m)$. To calculate these the B_m and H_m values at the $(BH)_{max}$ point must be known. Alternatively, if the three points corresponding to the remanent flux density B_r, the $(BH)_{max}$ and the coercive force H_c are given, the working values can be calculated from

$$B_m = \sqrt{[(BH)_{max}(B_r/H_c)]} \quad \text{and} \quad H_m = \sqrt{[(BH)_{max}(H_c/B_r)]}$$

2.4.2 Magnetomechanical effects

Mechanical forces are developed in magnetic field systems in such a way that the resulting movement increases the flux linkage with the electric circuit, or lowers the m.m.f. required for a given flux. In the former case an increase of linkage requires more energy from the circuit, making mechanical energy available; in the latter, stored magnetic energy is released in mechanical form.

Systems in which magnetomechanical forces are developed are shown diagrammatically in *Figure 2.15*.

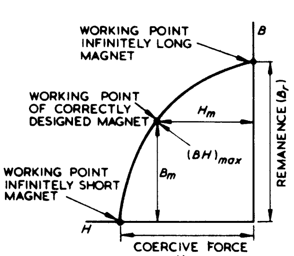


Figure 2.14 Ideal permanent magnet conditions

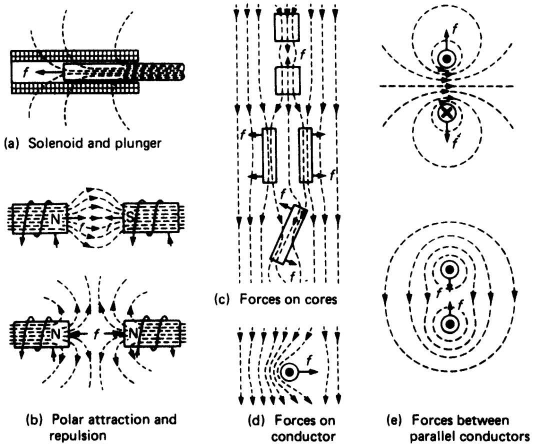


Figure 2.15 Magnetomechanical forces

(*a*) *Solenoid and magnetic core*: The core is drawn into the solenoid, increasing the magnetic flux and, in consequence, the circuit's flux linkages.

(*b*) *Attraction or repulsion of magnetised surfaces*: The attraction in (*b*) increases the flux by reducing the reluctance of the intergap. Repulsion gives more space for the opposing fluxes and again reduces the reluctance.

(*c*) *Forces on magnetic cores in a magnetic field*: Cores in line with the field attract each other, cores side by side repel each other, and a core out of line with the general field direction experiences a force tending to align it.

(*d*) *Electromagnetic force on current-carrying conductor*: A current-carrying conductor lying in an externally produced (or 'main') field tends to move so as to increase the flux on that side where its own field has the same direction as the main field.

(*e*) *Electromagnetic force between current-carrying conductors*: Two parallel conductors carrying currents in opposite directions repel, for in moving apart they provide a greater area for the flux between them. If carrying currents in the same direction, they attract, tending to provide a shorter path for the common flux.

It is worth noting as a guide to the behaviour of magnetic field problems (although not a physical explanation of that behaviour) that the forces observed are in directions such as would cause the flux lines to shorten their length and to expand laterally, as if they were stretched elastic threads.

The calculation of the force developed in the cases is based on the movement of the force-system by an amount dl and the amount of mechanical energy dW thereby absorbed or released. Then the force is

$$f = \mathrm{d}W/\mathrm{d}l$$

With energy in joules and displacement in metres, the force is given in newtons. The calculation is only directly possible in a few simple cases, as below. The references are to the diagrams in *Figure 2.15*.

Case (*c*): The energy stored per cubic metre of a medium in which a magnetising force H produces a density B is $\frac{1}{2}BH = \frac{1}{2}B^2/\mu_r\mu_0$ joules. At an iron surface in air, a movement of the surface into a space originally occupied by air results, for a constant density B, in a reduction of the energy per cubic metre from $B^2/2\mu_0$ to $B^2/2\mu_r\mu_0$, since for air $\mu_r = 1$. The force must therefore be

$$f = \Leftarrow\frac{B^2}{2\mu_0}\left(1-\frac{1}{\mu_r}\right)\Bigg] \simeq \Leftarrow\frac{B^2}{2\mu_0} \text{ newtons per square metre}$$

the latter expression being sufficiently close when $\mu_r \gg 1$.

Case (*d*): This case is of particular importance, as it is the basic principle of normal motors and generators, and of moving-coil permanent magnet instruments. Consideration of the mechanical energy gives, for a current I of length l lying perpendicular to the main field, a force

$$f = BlI$$

where B is the flux density. The force, as indicated in *Figure 2.15*(*d*), is at right angles to B and to I. Suppose the conductor to be moved in the direction of the force (either with it or against it): the work done in a displacement of x is

$$fx = W = BlIx = \Phi I \text{ joules}$$

where $\Phi = Blx$ is the total flux cut across by the conductor.

2.4.3 Electromagnetic induction

A magnetic field is a store of energy. When it is increased or decreased, the amount of stored energy increases or decreases. Where the energy is obtained from, or restored to, an associated electric circuit, the energy delivered or received is in the form of a current flowing by reason of an *induced e.m.f.* for a time (specified by the conditions), these three being the essential associated quantities determining electrical energy. The relative directions of e.m.f. and current depend on the direction of energy flow. This is described by *Lenz's law* (*Figure 2.16*), which states that the direction of the e.m.f. induced by a change of linked magnetic field is such as would oppose the change if allowed to produce a current in the associated circuit.

Faraday's law states that the e.m.f. induced in a circuit by the linked magnetic field is proportional to the rate of change of flux linkage with time. The flux linkage is the summation of products of magnetic flux with the number of turns of the circuit linked by it. Then

$$e = -\mathrm{d}\psi/\mathrm{d}t = -\Sigma N(\mathrm{d}\Phi/\mathrm{d}t)\Leftarrow$$

the negative sign being indicative of the direction of the e.m.f. as specified in Lenz's law.

Consider a circuit of N turns linked completely with a flux Φ. The linkage $= \Phi N$ may change in a variety of ways.

(1) Supposing the flux is constant in value, the circuit may move through the flux (relative motion of flux and circuit: the *motional* or *generator effect*).
(2) Supposing the coil is stationary with reference to the magnetic path of the flux, the latter may vary in magnitude (flux pulsation: the *pulsational* or *transformer effect*).
(3) Both changes may occur simultaneously (movement of coil through varying flux: combination of the effects in (1) and (2)).

The generator effect is associated with conversion of energy between the electrical and mechanical form, using an intermediate magnetic form; the transformer effect concerns the conversion of electrical energy into or from magnetic energy.

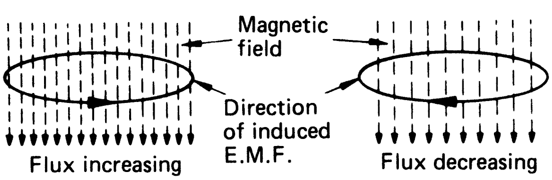


Figure 2.16 Faraday–Lenz law

2.4.3.1 *Generator effect*

In simplified terms applicable to heteropolar rotating electrical machines (*Figure 2.17*) the instantaneous e.m.f. due to rate of change of linkage resulting from the motion at speed u of an N-turn full-pitch coil of effective length l is $e = 2NBlu$, where B is the flux density in which the coil sides move at the instant considered.

On this expression as a basis, well-known formulae for the motional e.m.f.s of machines can be derived. For example, consider the arrangement (right) in *Figure 2.17*, where the flux density B is considered to be uniform: let the coil rotate at angular velocity ω_r rad/s corresponding to a speed $n = \omega_r/2\pi\psi$rev/s. Then the peripheral speed of the coil is $u = \omega_r R$ if its radius is R. Let the coil occupy a position perpendicular to the flux axis when time $t = 0$. At $t = \theta/\omega_r$ it will be in a position making the angle θ. Its rate of moving across the flux is $\omega_r R \sin\theta$, and the instantaneous coil e.m.f. is

$$e = \omega_r RNBl \sin\theta\psi = \omega_r N\Phi_m \sin\theta\psi$$

where $\Phi_m = 2BlR$ is the maximum flux embraced, i.e. at $\theta = 0$. The e.m.f. is thus a sine function of frequency $\omega_r/2\pi\psi$ and the values peak:

$$e_m = \omega_r N\Phi_m \quad \text{r.m.s.;} \quad E = (1/\sqrt{2})\omega_r N\Phi_m$$

The same result is obtained by a direct application of the Faraday law. At $t = 0$ the linked flux is Φ_m; at $t = \theta/\omega_r$ it is $\Phi_m \cos\theta = \Phi_m \cos\omega_r t$. The instantaneous e.m.f. is

$$e = -\mathrm{d}\psi/\mathrm{d}t = -N\Phi_m \mathrm{d}(\cos\omega_r t)/\mathrm{d}t = \omega_r N\Phi_m \sin\theta\psi$$

as before.

2.4.3.2 *Transformer effect*

The practical case concerns a coil of N turns embracing a varying flux Φ. If the flux changes sinusoidally with time it can be expressed as

$$\Phi = \Phi_m \cos\omega t = \Phi_m \cos 2\pi f t$$

where Φ_m is its time maximum value, f is its frequency, and $\omega = 2\pi f$ is its angular frequency. The instantaneous e.m.f. in the coil is

$$e = -N(d\Phi/\mathrm{d}t) = \omega N\Phi_m \sin\omega t$$

This relation forms the basis of the e.m.f. induced in transformers and induction motors. The e.m.f. and flux relationship is that of *Figures 2.16* and *2.21*(*c*).

2.4.3.3 *Calculation of induced e.m.f.*

The two methods of calculating electromagnetically induced e.m.f.s are: (1) the change-of-flux law, and (2) the flux-cutting law.

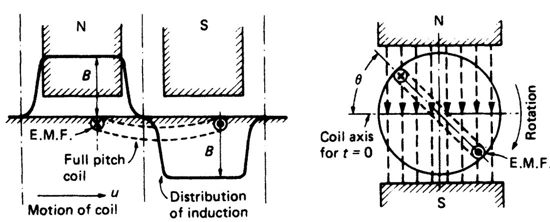


Figure 2.17 Motional e.m.f.

Flux change: This law has the basic form

$$e = -N(\mathrm{d}\Phi/\mathrm{d}t) \Leftarrow$$

and is applicable where a circuit of constant shape links a changing magnetic flux.

Flux cutting: Where a conductor of length l moves at speed u at right angles to a uniform magnetic field of density B, the e.m.f. induced in the conductor is

$$e = Blu$$

This can be applied to the motion of conductors in constant magnetic fields and when sliding contacts are involved.

Linkage change: Where coils move in changing fluxes, and both flux-pulsation and flux-cutting processes occur, the general expression.

$$e = -\mathrm{d}\psi/\mathrm{d}t$$

must be used, with variation of the linkage expressed as the result of both processes.

2.4.3.4 Constant linkage principle

The linkage of a *closed* circuit cannot be changed instantaneously, because this would imply an instantaneous change of associated magnetic energy, i.e. the momentary appearance of infinite power. It can be shown that the linkage of a closed circuit of *zero resistance* and no internal source cannot be changed at all. The latter concept is embodied in the following theorem.

Constant linkage theorem The linkage of a closed passive circuit of zero resistance is a constant. External attempts to change the linkage are opposed by induced currents that effectively prevent any net change of linkage.

The theorem is very helpful in dealing with transients in highly inductive circuits such as those of transformers, synchronous generators, etc.

2.4.3.5 Ideal transformer

An ideal transformer comprises two resistanceless coils embracing a common magnetic circuit of infinite permeability and zero core loss (*Figure 2.18*). The coils produce no leakage flux: i.e. the whole flux of the magnetic circuit completely links both coils. When the primary coil is energised by an alternating voltage V_1, a corresponding flux of peak value Φ_m is developed, inducing in the N_1-turn primary coil an e.m.f. $E_1 = V_1$. At the same time an e.m.f. E_2 is induced in the N_2-turn secondary coil. If the terminals of the secondary coil are connected to a load taking a current I_2, the primary coil must accept a balancing current I_1 such that $I_1N_1 = I_2N_2$, as the core requires zero excitation. The operating conditions are therefore

$$N_1/N_2 = E_1/E_2 = I_2/I_1; \quad \text{and} \quad E_1I_1 = E_2I_2$$

The secondary load impedance $Z_2 = E_2/I_2$ is reflected into the primary to give the impedance $Z_1 = E_1/I_1$ such that

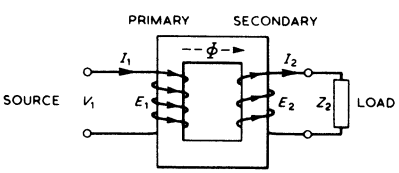


Figure 2.18 Ideal transformer

$$Z_1 = (N_1/N_2)^2 Z_2$$

A practical power transformer differs from the ideal in that its core is not infinitely permeable and demands an excitation $N_1I_0 = N_1I_1 - N_2I_2$; the primary and secondary coils have both resistance and magnetic leakage; and core losses occur. By treating these effects separately, a practical transformer may be considered as an ideal transformer connected into an external network to account for the defects.

2.4.3.6 Electromagnetic machines

An electromagnetic machine links an electrical energy system to a mechanical one, by providing a reversible means of energy flow between them in the common or 'mutual' magnetic flux linking stator and rotor. Energy is stored in the field and released as work. A current-carrying conductor in the field is subjected to a mechanical force and, in moving, does work and generates a counter e.m.f. Thus the force–motion product is converted to or from the voltage–current product representing electrical power.

The energy-rate balance equations relating the mechanical power p_e, and the energy stored in the magnetic field w_f, are:

Motor: $p_e = p_m = \mathrm{d}w_f/\mathrm{d}t$

Generator: $p_m = p_e + \mathrm{d}w_f/\mathrm{d}t$

The mechanical power term must account for changes in stored kinetic energy, which occur whenever the speed of the machine and its coupled mechanical loads alter.

Reluctance motors The force between magnetised surfaces (*Figure 2.15*(*b*)) can be applied to rotary machines (*Figure 2.19*(*a*)). The armature tends to align itself with the field axis, developing a *reluctance torque*. The principle is applied to miniature rotating-contact d.c. motors and synchronous clock motors.

Machines with armature windings Consider a machine rotating with constant angular velocity ω_r and developing a torque M. The mechanical power is $p_m = M\omega_r$: the electrical power is $p_e = ei$, where e is the counter e.m.f. due to the reaction of the mutual magnetic field. Then $ei = M\omega_r + \mathrm{d}w_f/\mathrm{d}t$ at every instant. If the armature conductor a in *Figure 2.19*(*b*) is running in a non-time-varying flux of local density B, the e.m.f. is entirely rotational and equal to $e_r = Blu = Bl\omega_rR$. The tangential force on the conductor is $f = Bli$ and the torque is $M = BliR$. Thus, $e_ri = M\omega_r$ because $\mathrm{d}w_f/\mathrm{d}t = 0$. This case applies to constant flux (d.c., three-phase synchronous and induction) machines.

If the armature in *Figure 2.19*(*b*) is given two conductors a and b they can be connected to form a turn. Provided the turn is of full pitch, the torques will always be additive. More turns in series form a winding. The total flux in the machine results from the m.m.f.s of all current-carrying conductors, whether on stator or rotor, but the torque arises from that component of the total flux at right angles to the m.m.f. axis of the armature winding.

Armature windings (*Figure 2.19*(*c*)) may be of the commutator or phase (tapped) types. The former is closed on itself, and current is led into and out of the winding by fixed brushes which include between them a constant number of conductors in each armature current path. The armature m.m.f. coincides always with the brush axis. Phase windings have separate external connections. If the winding is on the rotor, its current and m.m.f. rotate with it and the external connections must be made through slip-rings. Two (or three) such windings with two-phase (or three-phase) currents can produce a resultant m.m.f. that rotates with respect to the windings.

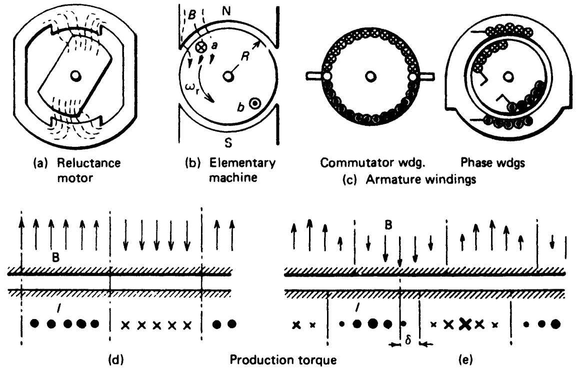


Figure 2.19 Electromagnetic machines

Torque Figure 2.19(d) shows a commutator winding arranged for maximum torque: i.e. the m.m.f. axis of the winding is displaced electrically $\pi/2$ from the field pole centres. If the armature has a radius R and a core length l, the flux has a constant uniform density B, and there are Z conductors in the $2p$ pole pitches each carrying the current I, the torque is $BRlIZ/2p$. This applies to a d.c. machine. It also gives the mean torque of a single-phase commutator machine if B and I are r.m.s. values and the factor $\cos\phi$ is introduced for any time phase angle between them.

The torque of a phase winding can be derived from *Figure 2.19(e)*. The flux density is assumed to be distributed sinusoidally, and reckoned from the pole centre to be $B_m \cos\alpha$. The current in the phase winding produces the m.m.f. F_a, having an axis displaced by angle $\delta\psi$ from the pole centre. The total torque is then

$$M = \pi B_m F_a l R \sin\delta\psi = \tfrac{1}{2}\pi\Phi F_a \sin\delta\psi$$

per pole pair. This case applies directly to the three-phase synchronous and induction machines.

Types of machine For unidirectional torque, the axes of the pole centres and armature m.m.f. must remain fixed relative to one another. Maximum torque is obtained if these axes are at right angles. The machine is technically better if the field flux and armature m.m.f. do not fluctuate with time (i.e. they are d.c. values): if they do alternate, it is preferable that they be co-phasal.

Workable machines can be built with (1) concentrated ('field') or (2) phase windings on one member, with (A) commutator or (B) phase windings on the other. It is basically immaterial which function is assigned to stator and which to rotor, but for practical convenience a commutator winding normally rotates. The list of chief types below gives the type of winding (1, 2, A, B) and current supply (d or a), with the stator first:

D.C. machine, 1d/Ad: The arrangement is that of *Figure 2.19(d)*. A commutator and brushes are necessary for the rotor.

Single-phase commutator machine, 1a/Aa: The physical arrangement is the same as that of the d.c. machine. The field flux alternates, so that the rotor m.m.f. must also alternate at the same frequency and preferably in time phase. Series connection of stator and rotor gives this condition.

Synchronous machine, Ba/1d: The rotor carries a concentrated d.c. winding, so the rotor m.m.f. must rotate with it at corresponding (synchronous) speed, requiring a.c. (normally three-phase) supply. The machine may be inverted (1d/Ba).

Induction machine, 2a/Ba (*Figure 2.19(e)*): The polyphase stator winding produces a rotating field of angular velocity ω_1. The rotor runs with a slip s, i.e. at a speed $\omega_1(1-s)$. The torque is maintained unidirectional by currents induced in the rotor winding at frequency $s\omega_1$. With d.c. supplied to the rotor (2a/Bd) the rotor m.m.f. is fixed relatively to the windings and unidirectional torque is maintained only at synchronous speed ($s=0$).

All electromagnetic machines are variants of the above.

2.4.3.7 Magnetohydrodynamic generator

Magnetohydrodynamics (m.h.d.) concerns the interaction between a conducting fluid in motion and a magnetic field. If a fast-moving gas at high temperature (and therefore ionised) passes across a magnetic field, an electric field is developed across the gaseous stream exactly as if it were a metallic conductor, in accordance with Faraday's law. The electric field gives rise to a p.d. between electrodes flanking the stream, and a current may be made to flow in an external circuit connected to the electrodes. The m.h.d. generator offers a direct conversion between heat and electrical energy.

2.4.3.8 Hall effect

If a flat conductor carrying a current I is placed in a magnetic field of density B in a direction normal to it (*Figure 2.20*), then an electric field is set up across the width of the conductor. This is the Hall effect, the generation of an e.m.f. by the movement of conduction electrons through the magnetic field. The Hall e.m.f. (normally a few microvolts) is picked off by tappings applied to the conductor edges, for the measurement of I or for indication of high-frequency powers.

2.4.4 Inductance

The e.m.f. induced in an electric circuit by change of flux linkage may be the result of changing the circuit's own current. A magnetic field always links a current-carrying circuit, and the linkage is (under certain restrictions) proportional to the current. When the current changes, the linkage also changes and an e.m.f. called the *e.m.f. of self-induction* is induced. If the linkage due to a current i in the circuit is $= \Phi N = Li$, the e.m.f. induced by a change of current is

$$e = -\mathrm{d}\psi/\mathrm{d}t) = -N(\mathrm{d}\Phi/\mathrm{d}t) = -L(\mathrm{d}i/\mathrm{d}t)$$

L is a coefficient giving the linkage per ampere: it is called the *coefficient of self-induction*, or, more usually, the *inductance*. The unit is the henry, and in consequence of its relation to linkage, induced e.m.f., and stored magnetic energy, it can be defined as follows.

A circuit has unit inductance (1 H) if: (a) the energy stored in the associated magnetic field is $\frac{1}{2}$ J when the current is 1 A; (b) the induced e.m.f. is 1 V when the current is changed at

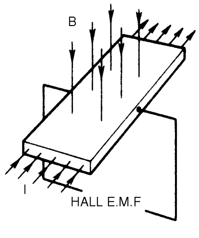


Figure 2.20 Hall effect

the rate 1 A/s; or (c) the flux linkage is 1 Wb-t when the current is 1 A.

2.4.4.1 Voltage applied to an inductor

Let an inductor (i.e. an inductive coil or circuit) devoid of resistance and capacitance be connected to a supply of constant potential difference V, and let the inductance be L. By definition (b) above, a current will be initiated, growing at such a rate that the e.m.f. induced will counterbalance the applied voltage V. The current must rise uniformly at V/L amperes per second, as shown in *Figure 2.21*(*a*), so long as the applied p.d. is maintained. Simultaneously the circuit develops a growing linked flux and stores a growing amount of magnetic energy. After a time t_1 the current reaches $I_1 = (V/L)t_1$, and has absorbed a store of energy at voltage V and average current $I_1/2$, i.e.

$$W_1 = V \cdot \tfrac{1}{2} I_1 \cdot t_1 = \tfrac{1}{2} V I_1 t_1 = \tfrac{1}{2} L I_1^2 \text{ joules}$$

since $V = I_1 L/t_1$. If now the supply is removed but the circuit remains closed, there is no way of converting the stored energy, which remains constant. The current therefore continues to circulate indefinitely at value I_1.

Suppose that V is applied for a time t_1, then reversed for an equal time interval, and so on, repeatedly. The resulting current is shown in *Figure 2.21*(*b*). During the first period t_1 the current rises uniformly to $I_1 = (V/L)t_1$ and the stored energy is then $\frac{1}{2}LI_1{}^2$. On reversing the applied voltage the current performs the same rate of change, but negatively so as to reduce the current magnitude. After t_1 it is zero and so is the stored energy, which has all been returned to the supply from which it came.

If the applied voltage is sinusoidal and alternates at frequency f, such that $v = v_m \cos 2\pi ft = v_m \cos \omega t$, and is switched on at instant $t = 0$ when $v = v_m$, the current begins to rise at rate v_m/L (*Figure 2.21*(*c*)); but the immediate reduction and subsequent reversal of the applied voltage require corresponding changes in the rate of rise or fall of the current. As $v = L(di/dt)$ at every instant, the current is therefore

$$i = \int \frac{v}{L}\, dt = \frac{v_m}{\omega L} \sin \omega t$$

The peak current reached is $i_m = v_m/\omega L$ and the r.m.s. current is $I = V/\omega L = V/X_L$, where $X_L = \omega L = 2\pi fL$ is the *inductive reactance*.

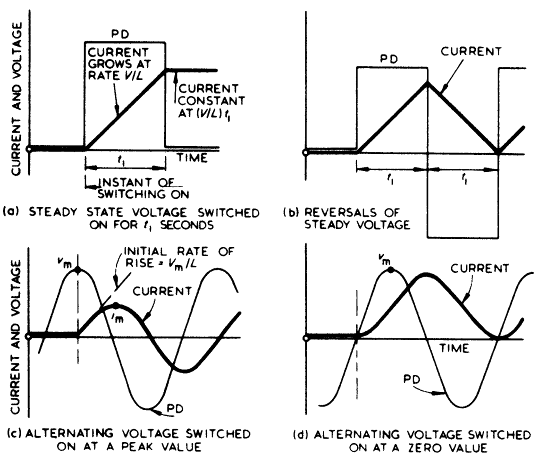


Figure 2.21 Voltage applied to a pure inductor

Should the applied voltage be switched on at a voltage zero (*Figure 2.21*(*d*)), the application of the same argument results in a sine-shaped current, unidirectional but pulsating, reaching the peak value $2i_m = 2v_m/\omega L$, or twice that in the symmetrical case above. This is termed the *doubling effect*. Compare with *Figure 2.21*(*b*).

2.4.4.2 Calculation of inductance

To calculate inductance in a given case (a problem capable of reasonably exact solution only in cases of considerable geometrical simplicity), the approach is from the standpoint of definition (c). The calculation involves estimating the magnetic field produced by a current of 1 A, summing the linkage ΦN produced by this field with the circuit, and writing the inductance as $L = \Sigma\Phi N$. The cases illustrated in *Figure 2.10* and *2.22* give the following results.

Long straight isolated conductor (*Figure 2.10*(*a*)) The magnetising force in a circular path concentric with the conductor and of radius x is $H = F/2\pi x = 1/2\pi x$; this gives rise to a circuital flux density $B_0 = \mu_0 H = \mu_0/2\pi x$. Summing the linkage from the radius r of the conductor to a distance s gives

$$= (\mu_0/2\pi) \ln(s/r) \Leftarrow$$

weber-turn per metre of conductor length. If s is infinite, so is the linkage and therefore the inductance: but in practice it is not possible so to isolate the conductor.

There is a magnetic flux following closed circular paths within the conductor, the density being $B_i = \mu x/2\pi r^2$ at radius x. The effective linkage is the product of the flux by that proportion of the conductor actually enclosed, giving $\mu/8\pi$ per metre length. It follows that the internal linkage produces a contribution $L_i = \mu/8\pi$ henry/metre, regardless of the conductor diameter on the assumption that the current is uniformly distributed. The absolute permeability μ of the conductor material has a considerable effect on the internal inductance.

Concentric cylindrical conductors (*Figure 2.10*(*b*)) The inductance of a metre length of concentric cable carrying equal currents oppositely directed in the two parts is due to the flux in the space between the central and the tubular conductor set up by the inner current alone, since the current in the outer conductor cannot set up internal flux. Summing the linkages and adding the internal linkage of the inner conductor:

$$L = (\mu/8\pi) + (\mu_0/2\pi) \ln(R/r) \text{ henry/metre}$$

Parallel conductors (*Figure 2.22*) Between two conductors (*a*) carrying the same current in opposite directions, the linkage is found by summing the flux produced by conductor

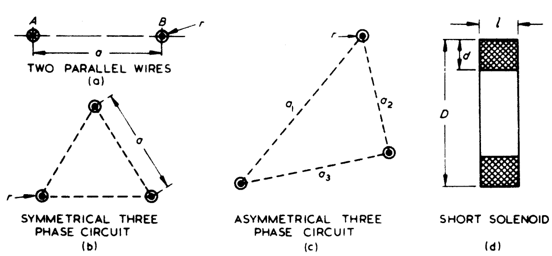


Figure 2.22 Parallel conductors

A in the space a assuming conductor B to be absent, and doubling the result. Provided that $a \gg r$, this gives for the loop

$$L = (\mu/4\pi) + (\mu_0/\pi)\ln(a/r) \text{ henry/metre}$$

It is permissible to regard one-half of the linkage as associated with each conductor, to give for each the *line-to-neutral* inductance

$$L_0 = (\mu/8\pi) + (\mu_0/2\pi)\ln(a/r)$$

A three-phase line (b) has the same line-to-neutral inductance if the conductors are symmetrically spaced. If, however, the spacing is asymmetric but the conductors are cyclically transposed (c), the expression applies with $a = \sqrt[3]{(a_1 a_2 a_3)}$, the geometric mean spacing.

Toroid (*Figure 2.10*(c)) For a core of permeability μ the inductance is

$$L = \mu N^2 A/2\pi R = \mu N^2 A/l \text{ henry}$$

where $l = 2\pi R$ is the mean circumference and A is the effective cross-sectional area of the core.

Solenoid A solenoid having a ratio length/diameter of at least 20 has an inductance approximating to that of the toroid above. A short solenoidal coil of overall diameter D, length l, radial thickness d, and N turns has an inductance given approximately by

$$L = \frac{6.4\mu_0 N^2 D^2}{3.5D + 8l} \cdot \frac{D - 2.25d}{D}$$

For the best ratio of inductance to resistance, $l = d$, giving a square winding cross-section, and $D = 4.7d$. Then $L = (0.8 \times 10^{-6})N^2 D$.

Inductor with ferromagnetic circuit (*Figure 2.10*(d)) Saturation makes it necessary to obtain the m.m.f. F for a series of magnetic circuit fluxes Φ. Then with an N-turn exciting coil the inductance is $L = N\Phi/I$, where $I = F/N$. Thus, L is a function of I, decreasing with increase of current. The variation can be mitigated by the inclusion in the magnetic circuit of an air gap to 'stiffen' the flux.

2.4.4.3 Mutual inductance

If two coils (primary and secondary) are so oriented that the flux developed by a current in one links the other, the two have mutual inductance. The pair have unit mutual inductance (1 H) if: (a) the energy stored in the common magnetic field is 1 J when the current in each circuit is 1 A; or (b) the e.m.f. induced in one is 1 V when the current in the other changes at the rate 1 A/s, or (c) the secondary linkage is 1 Wb-t when the primary current is 1 A.

Figure 2.23(a) shows two coils on a common magnetic circuit: it is assumed that all the flux due to a primary current also links the secondary (a condition approached in a transformer). Let 1 A in the two-turn primary produce 2 Wb. The primary self-inductance is consequently $L_1 = 2 \times 2 = 4$ Wb-t/A $= 4$ H. The secondary linkage is $4 \times 2 = 8$ Wb-t, so that the mutual inductance is $L_{21} = 8$ H. Let now a current of 1 A circulate instead in the secondary (b): it develops double the m.m.f. of that developed by the primary in (a) and twice the flux, i.e. 4 Wb. The self-inductance is $L_2 = 4 \times 4 = 16$ H. The primary linkage is $2 \times 4 = 8$, i.e. $L_{12} = 8$ H. Thus, $L_{12} = L_{21}$.

In *Figure 2.23*(c) the coils are connected in *series aiding*. With a current of 1 A, the total m.m.f. is $2 + 4 = 6$, the common flux is 6 Wb and the total inductance is $6 \times 6 = 36$ H, which can be shown to be

$$L = L_1 + L_2 + L_{12} + L_{21} = 4 + 16 + 2(8) = 36 \text{ H}$$

For the *series opposing* connection (d) the m.m.f.s oppose with a net value $4 - 2 = 2$, and the resulting flux is 2 Wb. The linkages oppose, amounting to $(4 \times 2) - (2 \times 2) = 4$. The total inductance is 4 H, obtained by

$$L = L_1 + L_2 - L_{12} - L_{21} = 4 + 16 - 2(8) = 4 \text{ H}$$

The example shows that $L_{12} = L_{21} = \sqrt{(L_1 L_2)}$. Normally the linkages are less complete, and the ratio $L_{12}/\sqrt{(L_1 L_2)} = k$, the *coefficient of coupling*.

2.4.4.4 Connection of inductors

In the absence of mutual inductance, the total inductance of a circuit consisting of inductors L_1, $L_2 \ldots$, is $L = L_1 + L_2 + \cdots$, if they are in series, and $L = 1/[(1/L_1) + (1/L_2) + \cdots]$ if they are in parallel. When there is mutual inductance, it is necessary to set up a circuit equation including the mutual inductance coefficients L_{12}, L_{13}, $L_{23} \ldots$. For two inductors the inductance, as already discussed, is

$$L = L_1 + L_2 \pm 2L_{12}$$

With coils associated on a common ferromagnetic circuit, L_{12} may differ from L_{21} because of saturation effects.

2.5 Electric field effects

When two conductors are separated by a dielectric medium and are maintained at a potential difference, an electric field exists between them. Consider two such conductors A and B (*Figure 2.24*): the application of a p.d. causes a transfer of conduction electrons from A to B, leaving A positive and making B negative, and setting up the electric field

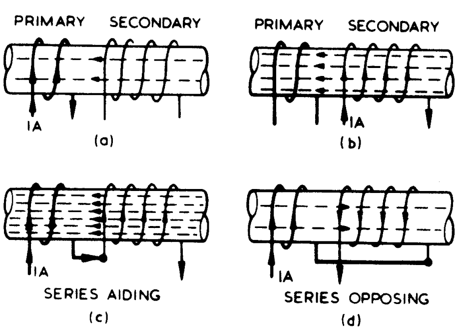


Figure 2.23 Mutual inductance

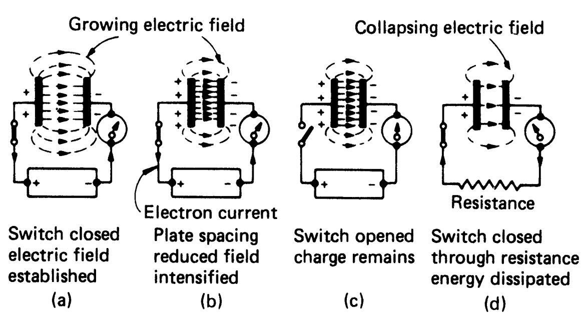


Figure 2.24 Capacitor charge and discharge

(*Figure 2.24*(*a*)). The positive charge on A prevents more than a given number of electrons leaving this conductor, depending on the p.d., the size and configuration, and the spacing. Similarly, the surplus of electrons on B repels others arriving, so that here, too, an equilibrium is established. If, as in (*b*), the spacing is reduced, a further electron transfer takes place until equilibrium is again reached; the charge and the electric field have been intensified by the increase in *capacitance*. If now the switch is opened, the charge, p.d. and field remain as a store of electric field energy (*c*). Let the supply be removed (*d*) and the switch closed through a resistor; the charges on the conductors are dissipated by an electron current from B to A, and the field energy is converted into heat in the resistor.

2.5.1 Electrostatics

The lines used to depict the pattern of an electric field begin on a positive charge and terminate on an equal negative one. Two similar point charges of q_1 and q_2 coulombs, spaced d metres apart in free space (or air) develop a force

$$f = q_1 q_2 / 4\pi\epsilon_0 d^2 \text{ newtons}$$

of repulsion if the charges have the same polarity, of attraction if they have opposite polarity, ϵ_0 being the electric space constant. The force on q_2 can be considered as due to its immersion in the electric field E_1 of q_1, i.e. $f = E_1 q_2$; whence

$$E_1 = q_2 / 4\pi\epsilon_0 d^2 \text{ volts per metre}$$

defining the electric field strength at distance d from a concentrated charge q_1. Thus, a unit charge (1 C) is that which repels a similar charge at unit distance (1 m) with a force of $1/4\pi\epsilon_0$ newton. Similarly, a field of unit strength (1 V/m) produces a mechanical force of 1 N on a unit charge placed in it.

To charge a system like that in *Figure 2.24*, a quantity of electricity has been moved under an applied electric force—i.e. work has been done measured by the charge transferred and the p.d. Unit p.d. (1 V) exists between two points in an electric field when unit work (1 J) is done in moving unit charge (1 C) between them. The two conductors are equipotential surfaces, and potential levels or equipotential lines can be drawn at right angles to the field lines (*Figure 2.25*). Equipotential lines resemble contour lines on the map of a hill: the closer they are, the greater is the voltage gradient. The change of potential in a given direction is

$$V = -\int E \cdot \mathrm{d}x$$

where E is the electric field strength, or potential gradient, in the direction x; whence $E = -\,\mathrm{d}V/\mathrm{d}x$.

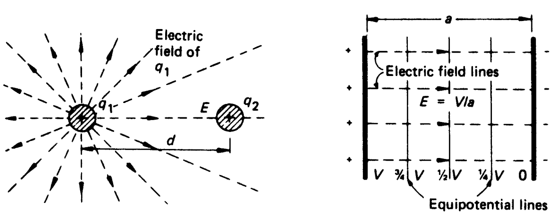


Figure 2.25 Electric fields

2.5.2 Capacitance

The configuration and geometry of the conductor system in *Figure 2.24* depends on the relation between the charge q on each conductor and the p.d., V, between them. Then $q = CV$, where C is the capacitance of the *capacitor* formed by the system. A capacitor has unit capacitance (1 F) if (a) the energy stored in the associated electric field is $\frac{1}{2}$ J for a p.d. of 1 V, or (b) the p.d. is 1 V for a charge of 1 C. Definition (a) follows from the energy storage property: if the p.d. across a capacitor is raised uniformly from zero to V_1, a charge $q_1 = CV_1$ is established at an average p.d. $\frac{1}{2}V_1$, so that the energy input is

$$W = \tfrac{1}{2}V_1 \cdot q_1 = \tfrac{1}{2}V_1 \cdot CV_1 = \tfrac{1}{2}CV_1^2$$

2.5.2.1 Dielectrics

Up to this point it has been assumed that the electric field between the plates of a capacitor has been established through vacuous space. If a material insulator is used—gas, liquid or solid—the electric field will exist therein. It will act on the molecules of the *dielectric* material in accordance with electrostatic principles to 'stretch' or 'rotate' them, and so to orientate the positive and negative molecular charges in opposite directions. This *polarisation* of the dielectric may be imagined to take place as in *Figure 2.26*. Before the p.d. is applied, the molecules of the dielectric material are neutral and unstrained. As the p.d. is raised from zero as in (*a*), the electric field acts to separate the positive and negative elements, the small charge displacement forming a *polarisation* current.

The effect of the application of a p.d. to a capacitor with a material dielectric, then, is to displace a surface charge q_d of polarisation, having a polarity opposite to that of the adjacent capacitor plate q_c. The electric field in the dielectric is due to the resultant or net charge $q = q_c - q_d$. The field strength (and therefore the p.d.) is less than would be expected for the charge q_c on the plates: the relative reduction is found to be approximately constant for a given dielectric. It is called the *relative permittivity*, symbol ϵ_r. The same field strength as for a vacuum will exist in the dielectric for ϵ_r times as much charge on the capacitor plates, so that the capacitor has ϵ_r times the capacitance of a similar capacitor having free space between its plates.

Permittivity effects can thus be taken into account either by assigning to a dielectric a relative permittivity or by considering its polarisation. The latter is of use where internal dielectric forces are concerned, and in dielectric breakdown (*Figure 2.26*(*b*)).

2.5.2.2 Calculation of capacitance

The capacitance of capacitors of simple geometry can be found by assigning, respectively, charges of +1 C and −1 C to the plates or other electrodes, between which the total

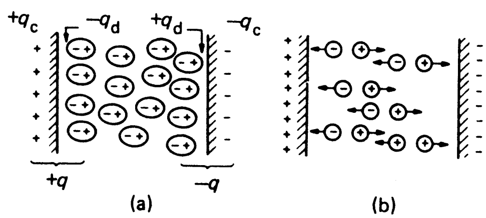


Figure 2.26 Dielectric polarisation and breakdown

electric flux is 1 C. From the field pattern the electric flux density D at any point is found. Then the electric field strength at the point is $E = D/\epsilon$, where $\epsilon = \epsilon_r\epsilon_0$ is the absolute permittivity of the insulating medium in which the electric flux is established. Integration of E over any path from one electrode to the other gives the p.d. V, whence the capacitance is $C = 1/V$.

Parallel plates (*Figure 2.27*(*a*)) The electric flux density is uniform except near the edges. By use of a *guard ring* maintained at the potential of the plate that it surrounds, the capacitance of the inner part is calculable on the reasonable assumption of uniform field conditions. With a charge of 1 C on each plate, and plates of area S spaced a apart, the electric flux density is $D = 1/S$, the electric field intensity is $E = D/\epsilon = 1/S\epsilon$, the potential difference is $V = Ea = a/S\epsilon$, and the capacitance is therefore

$$C = q/V = \epsilon(S/a)\Leftarrow$$

A case of interest is that of a parallel plate arrangement (*Figure 2.27*(*b*)), with two dielectric materials, of thickness a_1 and a_2 and absolute permittivity ϵ_1 and ϵ_2, respectively. The voltage gradient is inversely proportional to the permittivity, so that $E_1\epsilon_1 = E_2\epsilon_2$. The field pattern makes it evident that the difference in polarisation produces an interface charge, but in terms of the charge q_c on the plates themselves the electric flux density is constant throughout. The total voltage between the plates is $V = V_1 + V_2 = E_1a_1 + E_2a_2$, from which the total capacitance can be obtained.

Concentric cylinders (*Figure 2.27*(*c*)) With a charge of 1 C per metre length, the electric flux density at radius x is $1/2\pi x$, whence $E_x = 1/2\pi x\epsilon$. Integrating for the p.d. gives

$$V = (1/2\pi\epsilon)\ln(R/r)\Leftarrow$$

The capacitance is consequently

$$C = 2\pi\epsilon/\ln(R/r) \text{ farad/metre}$$

The electric field strength (voltage gradient) E is inversely proportional to the radius, over which it is distributed hyperbolically. The maximum gradient occurs at the surface of the inner conductor and amounts to

$$E_m = V/r\ln(R/r)\Leftarrow$$

At any other radius x, $E_x = E_m(r/x)$. For a given p.d. V and gradient E_m there is one value of r to give minimum overall radius R: this is

$$r = V/E_m \text{ and } R = 2.72r$$

For the cylindrical capacitor (*d*) with two dielectrics, of permittivity ϵ_1 between radii r and ρ and ϵ_2 between ρ and R, the maximum gradients are related by $E_{m1}\epsilon_1 r = E_{m2}\epsilon_2\rho$.

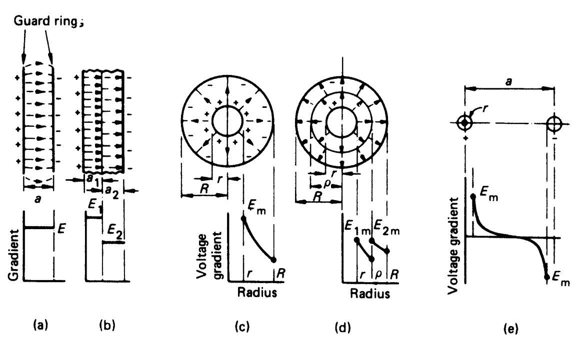


Figure 2.27 Capacitance and voltage gradient

Parallel cylinders (*Figure 2.27*(*e*)) The calculation leads to the value

$$C = \pi\epsilon/\ln(a/r) \text{ farad/metre}$$

for the capacitance between the conductors, provided that $a \gg r$. It can be considered as composed of two series-connected capacitors each of

$$C_0 = 2\pi\epsilon/\ln(a/r) \text{ farad/metre}$$

C_0 being the *line-to-neutral* capacitance. A three-phase line has a line-to-neutral capacitance identical with C_0, the interpretation of the spacing a for transposed asymmetrical lines being the same as for their inductance.

The voltage gradient of a two-wire line is shown in *Figure 2.27*(*e*). If $a \gg r$, the gradient in the immediate vicinity of a wire may be taken as due to the charge thereon, the further wire having little effect: consequently,

$$E_m = V/r\ln(a/r)\Leftarrow$$

is the voltage gradient at a conductor surface.

2.5.2.3 *Connection of capacitors*

If a bank of capacitors of capacitance C_1, C_2, $C_3\ldots$, be connected in *parallel* and raised in combination each to the p.d. V, the total charge is the sum of the individual charges VC_1, VC_2, $VC_3\ldots$, whence the total combined capacitance is

$$C = C_1 + C_2 + C_3 + \cdots\Leftarrow$$

With a *series* connection, the same displacement current occurs in each capacitor and the p.d. V across the series assembly is the sum of the individual p.d.s:

$$\begin{aligned} V &= V_1 + V_2 + V_3 + \cdots\Leftarrow \\ &= q[(1/C_1) + (1/C_2) + (1/C_3) + \cdots]\Leftarrow \\ &= q/C \end{aligned}$$

so that the combined capacitance is obtained from

$$C = 1/[(1/C_1) + (1/C_2) + (1/C_3) + \cdots]\Leftarrow$$

2.5.2.4 *Voltage applied to a capacitor*

The basis for determining the conditions in a circuit containing a capacitor to which a voltage is applied is that the p.d. v across the capacitor is related definitely by its capacitance C to the charge q displaced on its plates: $q = Cv$.

Let a direct voltage V be suddenly applied to a circuit devoid of all characteristic parameters except that of capacitance C. At the instant of its application, the capacitor must accept a charge $q = Cv$, resulting in an infinitely large current flowing for a vanishingly short time. The energy stored is $W = \frac{1}{2}Vq = \frac{1}{2}CV^2$ joules. If the voltage is raised or lowered uniformly, the charge must correspondingly change, by a constant charging or discharging current flowing during the change (*Figure 2.28*(*a*)).

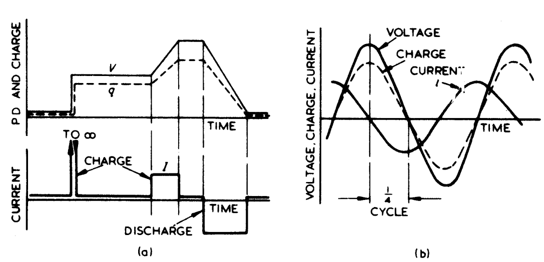


Figure 2.28 Voltage applied to a pure capacitor

If the applied voltage is sinusoidal, as in (*b*), such that $v = v_m \cos 2\pi ft = v_m \cos \omega t$, the same argument leads to the requirement that the charge is $q = q_m \cos \omega t$, where $q_m = Cv_m$. Then the current is $i = dq/dt$, i.e.

$$i = -\omega C v_m \sin \omega t$$

with a peak $i_m = \omega C v_m$ and an r.m.s. value $I = \omega CV = V/X_c$, where $X_c = 1/\omega C$ is the *capacitive reactance*.

2.5.3 Dielectric breakdown

A dielectric material must possess: (a) a high insulation resistivity to avoid leakage conduction, which dissipates the capacitor energy in heat; (b) a permittivity suitable for the purpose—high for capacitors and low for insulation generally; and (c) a high electric strength to withstand large voltage gradients, so that only thin material is required. It is rarely possible to secure optimum properties in one and the same material.

A practical dielectric will break down (i.e. fail to insulate) when the voltage gradient exceeds the value that the material can withstand. The breakdown mechanism is complex.

2.5.3.1 Gases

With gaseous dielectrics (e.g. air and hydrogen), ions are always present, on account of light, heat, sparking, etc. These are set in motion, making additional ionisation, which may be cumulative, causing glow discharge, sparking or arcing unless the field strength is below a critical value. A field strength of the order of 3 MV/m is a limiting value for gases at normal temperature and pressure. The dielectric strength increases with the gas pressure.

The polarisation in gases is small, on account of the comparatively large distances between molecules. Consequently, the relative permittivity is not very different from unity.

2.5.3.2 Liquids

When very pure, liquids may behave like gases. Usually, however, impurities are present. A small proportion of the molecules forms positive or negative ions, and foreign particles in suspension (fibres, dust, water, droplets) are prone to align themselves into semiconducting filaments: heating produces vapour, and gaseous breakdown may be initiated. Water, because of its exceptionally high permittivity, is especially deleterious in liquids such as oil.

2.5.3.3 Solids

Solid dielectrics are rarely homogeneous, and are often hygroscopic. Local space charges may appear, producing absorption effects; filament conducting paths may be present; and local heating (with consequent deterioration) may occur. Breakdown depends on many factors, especially thermal ones, and is a function of the time of application of the p.d.

2.5.3.4 Conduction and absorption

Solid dielectrics in particular, and to some degree liquids also, show conduction and absorption effects. Conduction appears to be mainly ionic in nature. Absorption is an apparent storing of charge *within* the dielectric. When a capacitor is charged, an initial quantity is displaced on its plates due to the *geometric* capacitance. If the p.d. is maintained, the charge gradually grows, owing to *absorptive* capacitance, probably a result of the slow orientation of permanent dipolar molecules. The current finally settles down to a small constant value, owing to conduction.

Absorptive charge leaks out gradually when a capacitor is discharged, a phenomenon observable particularly in cables after a d.c. charge followed by momentary discharge.

2.5.3.5 Grading

The electric fields set up when high voltages are applied to electrical insulators are accompanied by voltage gradients in various parts thereof. In many cases the gradients are anything but uniform: there is frequently some region where the field is intense, the voltage gradient severe and the dielectric stress high. Such regions may impose a controlling and limiting influence on the insulation design and on the working voltage. The process of securing improved dielectric operating conditions is called *grading*. The chief methods available are:

(1) The avoidance of sharp corners in conductors, near which the gradient is always high.
(2) The application of high-permittivity materials to those parts of the dielectric structure where the stress tends to be high, on the principle that the stress is inversely proportional to the permittivity: it is, of course, necessary to correlate the method with the dielectric strength of the material to be employed.
(3) The use of intersheath conductors maintained at a suitable intermediate potential so as to throw less stress on those parts which would otherwise be subjected to the more intense voltage gradients.

Examples of (1) are commonly observed in high-voltage apparatus working in air, where large rounded conductors are employed and all edges are given a large radius. The application of (2) is restricted by the fact that the choice of materials in any given case is closely circumscribed by the mechanical, chemical and thermal properties necessary. Method (3) is employed in capacitor bushings, in which the intersheaths have potentials adjusted by correlation of their dimensions.

2.5.4 Electromechanical effects

Figure 2.29 summarises the mechanical force effects observable in the electric field. In (*a*), (*b*) and (*c*) are sketched the field patterns for cases already mentioned in connection with the laws of electrostatics. The surface charges developed on high-ϵ_r materials are instrumental in producing the forces indicated in (*d*). Finally, (*e*) shows the forces on pieces of dielectric material immersed in a gaseous or liquid insulator and subjected to a non-uniform electric field. The force

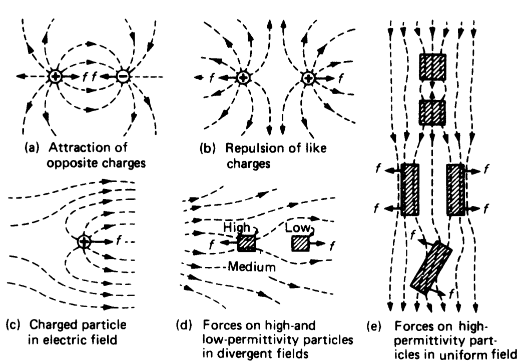


Figure 2.29 Electromechanical forces

direction depends upon whether the piece has a higher or lower permittivity than the dielectric medium in which it lies. Thus, pieces of high permittivity are urged towards regions of higher electric field strength.

2.6 Electromagnetic field effects

Electromagnetic field effects occur when electric charges undergo acceleration. The effects may be negligible if the rate of change of velocity is small (e.g. if the operating frequency is low), but other conditions are also significant, and in certain cases effects can be significant even at power frequencies.

2.6.1 Movement of charged particles

Particles of small mass, such as electrons and protons, can be accelerated in vacuum to very high speeds.

Static electric field The force developed on a particle of mass m carrying a positive charge q and lying in an electric field of intensity (or gradient) E is $f = qE$ in the direction of E, i.e. from a high-potential to a low-potential region (*Figure 2.30*(*a*)). (If the charge is negative, the direction of the force is reversed.) The acceleration of the particle is $a = f/m = E(q/m)$; and if it starts from rest its velocity after time t, is $u = at = E(q/m)t$. The kinetic energy $\frac{1}{2}mu^2$ imparted is equal to the change of potential energy Vq, where V is the p.d. between the starting and finishing points in the electric field. Hence, the velocity attained from rest is

$$u = \sqrt{[2V(q/m)]} \Leftarrow$$

For an electron ($q = -1.6 \times 10^{-19}$ C, $m_0 = 0.91 \times 10^{-30}$ kg) falling through a p.d. of 1 V the velocity is 600 km/s and the kinetic energy is $w = Vq = 1.60 \times 10^{-19}$ J, often called an electron-volt, 1 eV.

If $V = 2.5$ kV, then $u = 30\,000$ km/s; but the speed cannot be indefinitely raised by increasing V, for as u approaches $c = 300\,000$ km/s, the free-space electromagnetic wave velocity, the effective mass of the particle begins to acquire a rapid relativistic increase to

$$m = m_0/[1 - (u/c)^2] \Leftarrow$$

compared with its 'rest mass' m_0.

Static magnetic field A charge q moving at velocity u is a current $i = qu$, and is therefore subject to a force if it moves across a magnetic field. The force is at right angles to u and to B, the magnetic flux density, and in the simple case of *Figure 2.30*(*b*) we have $f = quB = ma = mu^2/R$, the particle being constrained by the force to move in a circular path of radius $R = (u/B)(m/q)$. For an electron $R = 5.7 \times 10^{-22}$ (u/B).

Combined electric and magnetic fields The two effects described above are superimposed. Thus, if the E and B fields are coaxial, the motion of the particle is helical.

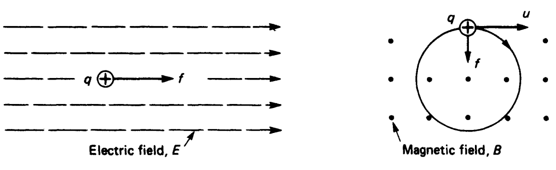


Figure 2.30 Motion of charged particles

The influence of static (or quasi-static) fields on charged particles is applied in cathode ray oscilloscopes and accelerator machines.

2.6.2 Free space propagation

In Section 1.5.3 the Maxwell equations are applied to propagation of a plane electromagnetic wave in free space. It is shown that basic relations hold between the velocity u of propagation, the electric and magnetic field components E and H, and the electric and magnetic space constants ϵ_0 and μ_0. The relations are:

$$u = 1/\sqrt{(\mu_0 \epsilon_0)} \simeq 3 \times 10^8 \text{ metre/second}$$

is the free space propagation velocity. The electric and magnetic properties of space impose a relation between E (in volts per metre) and H (in amps per metre) given by

$$E/H = \sqrt{(\mu_0/\epsilon_0)} = 377\,\Omega.\psi$$

called the *intrinsic impedance* of space. Furthermore, the energy densities of the electric and magnetic components are the same, i.e.

$$\tfrac{1}{2}\epsilon_0 E^2 = \tfrac{1}{2}\mu_0 H^2$$

Propagation in power engineering is not (at present) by space waves but by guided waves, a conducting system being used to direct the electromagnetic energy more effectively in a specified path. The field pattern is modified (although it is still substantially transverse), but the essential physical propagation remains unchanged. Such a guide is called a transmission line, and the fields are normally specified in terms of the inductance and capacitance properties of the line configuration, with an effective impedance $z_0 = \sqrt{(L/C)}$ differing from 377 Ω.

2.6.3 Transmission line propagation (see also Section 36)

If the two wires of a long transmission line, originally dead, are suddenly connected to a supply of p.d. v, an energy wave advances along the line towards the further end at velocity u (*Figure 2.31*). The wave is characterised by the fact that the advance of the voltage charges the line capacitance, for which an advancing current is needed: and the advance of the current establishes a magnetic field against a counter-e.m.f., requiring the voltage for maintaining the advance. Thus, current and voltage are propagated simultaneously. Let losses be neglected, and L and C be the inductance and capacitance per unit length of line. In a brief time interval dt, the waves advance by a distance $u \cdot \mathrm{d}t$. The voltage is established across a capacitance $Cu \cdot \mathrm{d}t$ and the rate of charge, or current, is

$$i = vCu \cdot \mathrm{d}t/\mathrm{d}t = vCu$$

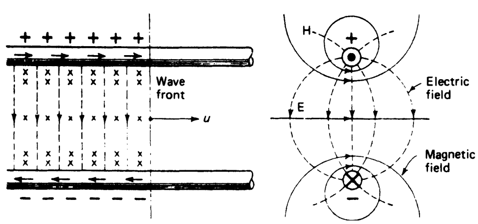


Figure 2.31 Transmission-line field

The current is established in an inductance $Lu \cdot \mathrm{d}t$ producing the magnetic linkages $iLu \cdot \mathrm{d}t$ in time dt, and a corresponding counter-e.m.f. overcome by

$$v = iLu \cdot \mathrm{d}t/\mathrm{d}t = iLu$$

These two expressions, by simple manipulation, yield

Propagation velocity $u = 1/\sqrt{(LC)}$⇐
Surge impedance $z_0 = v/i = \sqrt{(L/C)}$⇐
Energy components $\frac{1}{2}Li^2 = \frac{1}{2}Cv^2$

For a line in air consisting of parallel conductors of radius r spaced a between centres, the inductance (neglecting internal linkage) and capacitance per unit length are

$$L = (\mu_0/\pi)\ln(a/r); \quad C = \pi\varepsilon_0/\ln(a/r) \Leftarrow$$

whence $u = 1/\sqrt{(\mu_0\epsilon_0)} \simeq 3 \times 10^8$ m/s, exactly as for free space propagation. For lines in which the relative permittivity (and/or, rarely, the relative permeability) of the medium conveying the electromagnetic wave is greater than unity, the speed is reduced by the factor $1/\sqrt{(\mu_r\epsilon_r)}$. Line loss and internal linkage slightly reduce the speed of propagation. With the L and C values quoted, the surge impedance is

$$z_0 = 120\ln(a/r) \Leftarrow$$

which is usually in the range 300–600 Ω. For cables the different geometry and the relative permittivity give a much lower value.

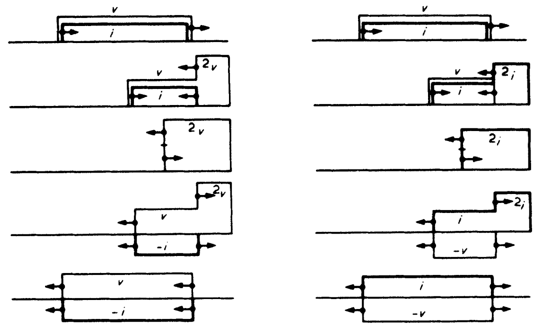
2.6.3.1 Reflection of surges

The relation of p.d. and current direction in a pair of wires forming a long transmission line is determined by the direction in which the energy is travelling (*Figure 2.31*). The current flows in the direction of propagation in the positive conductor, and returns in the negative. Two waves travelling in opposite directions on a line must have *either* the currents *or* the p.d.s in opposite senses. If two such waves meet, either the currents are subtractive and the voltage additive, or the reverse. In each case the natural ratio $v/i = z_0$ for each wave is not apparent: in fact, if the resultant voltage/current ratio is not z_0, the actual distribution of current and voltage must be due to two component waves having opposite directions of propagation. Such conditions arise when a surge is *reflected* at the end of a line.

Consider a steep-fronted surge which reaches the *open-circuited* end of its guiding line. At the point the current must be zero, which requires an equal reflected surge current of opposite sense. The voltages have the same sense and combined to give a doubling effect (*Figure 2.32*(*a*)). Reversal of the energy flow imposed by the line discontinuity, i.e. reflection, is thus accompanied by voltage doubling and an elimination of current. In unit length of a surge, the total energy is $\frac{1}{2}Li^2 + \frac{1}{2}Cv^2$; when two surges (one incident, one reflected) are superimposed, the total energy is electrostatic and of value $\frac{1}{2}C(2v)^2 = 2Cv^2$, which, of course, is equal to the total energy per unit length of the two waves: $2(\frac{1}{2}Cv^2 + \frac{1}{2}Li^2) = Cv^2 + Li^2 = 2Cv^2$.

If the sending-end generator has zero impedance and remains closed on to the line, the returning surge is again reflected at voltage v and the to-and-fro 'oscillation' is maintained indefinitely. In practice, the presence of the losses reduces the reflections and the voltage settles down finally to the steady value v, with a current determined by line admittance.

An incident surge which reaches the *short-circuited* end of a line is presented with a condition of termination across which no p.d. can be maintained. The voltage collapses to zero, initiating a doubling of the current and reflection with reversed voltage (*Figure 2.32*(*b*)).



(a) REFLECTION FROM OPEN END (b) REFLECTION FROM SHORT-CIRCUITED END

Figure 2.32 Surge reflection

If the sending-end generator has zero impedance and remains closed on to the line, reflections take place repeatedly with an increase of i in the current each time, building up eventually to an infinite value—as would be expected in a lossless line with its end short circuited. Under realisable conditions the current rises with steps of reducing size to a value limited by the series line impedance.

Terminated line If a line is terminated on an impedance z, only partial reflection will take place, some of the incident surge energy being dissipated as heat in the resistive part of z. Let the incident surge be v_1i_1 such that $v_1/i_1 = z_0$; and let the reflected wave be v_2i_2, with $v_2/i_2 = -z_0$. The negative sign is required algebraically to take account of the reversal of either current or voltage by reflection. The voltage v and current i at the termination z during the reflection are such that $v = v_1 + v_2$ and $i = i_1 + i_2$. But $v = iz$, so that, for the reflected current and voltage,

$$v_2 = v_1\frac{z - z_0}{z + z_0} = \alpha v_1 \quad \text{and} \quad i_2 = -i_1\frac{z - z_0}{z + z_0}$$

where α is the *reflection factor*. At the termination itself

$$v = v_1\frac{2z}{z + z_0} = \beta v_1 \quad \text{and} \quad i = i_1\frac{2z_0}{z + z_0}$$

where $\beta = 1 + \alpha$ is the *absorption factor*. The characteristic impedance is resistive, and reflection takes place unless $z = z_0$ and is also resistive. In the latter case v_2 and i_2 vanish, there is no reflection, and the energy in the incident surge v_1i_1 is absorbed in z at the same rate as that at which it arrives.

Line junction or discontinuity A line having an abrupt change of surge impedance from z_0 to z_0' (as, for example, at the junction of a branch line, or at line-cable connection) can be treated as above, substituting z_0' for z. The voltage $v = \beta v_1$ is then characteristic of the *transmitted surge*, and β becomes the transmission factor.

Writing $v = 2v_1 \cdot z/(z + z_0)$, it is seen that v can be considered as the voltage across the load z of a voltage generator of e.m.f. $2v_1$ and internal impedance z_0. Any line termination of reasonable simplicity can now be dealt with. The discontinuity z may be a shunt load, or any combination of loads and extension lines reducible to an equivalent z. If z is resistive, the calculation of the terminal v and i is straightforward; but if z contains a time function (i.e. if it contains inductive and/or capacitive elements), the expressions

$v = \beta v_1$ and $v_2 = \alpha v_1$ become integro-differential equations, to be solved by the methods given in Chapter 5. For z resistive, reflection and transmission (or absorption) of surges takes place without change of shape: for z containing terms in L and C, the shape of the incident surge is modified.

Single reflections Applying the equivalent circuit, a line on open-circuit has $z = \infty$, giving $v = 2v_1$ and $v_2 = v_1$. For a short-circuited line, $z = 0$, $v = 0$ and $v_2 = -v_1$. These cases are shown in *Figure 2.32*.

Shunt inductor termination: Because L offers infinite opposition to infinite rate of current rise, it acts as an open circuit at the instant of surge arrival, degenerating with time constant L/z_0 to a condition of short-circuit. Thus the voltage at the termination is

$$v = 2v_1 \exp(-t \cdot z_0/L)$$

Shunt capacitor termination: C is initially the equivalent of a short circuit, but charges as the surge continues to arrive, and eventually approaches the charged state, for which it acts as an open circuit. The voltage across it is consequently

$$v = 2v_1[1 - \exp(-t/Cz_0)]$$

Series inductor termination: If an inductor is inserted into a line of surge impedance z_0, the equivalent circuit is made up of the generator of voltage $2v_1$, loaded with z_0, L and z_0 in series. The voltage transmitted into the line extension is then

$$v' = v_1[1 - \exp(-t \cdot 2z_0/L)]$$

showing the reduction of wavefront steepness produced. This roughly represents the action of a series surge modifier.

Multiple reflections Surges on power networks will be subject to repeated reflections at terminations, junctions, towers and similar discontinuities. Such cases are handled by developing the appropriate reflection and transmission (or absorption) factors α and β for each discontinuity and each direction. Procedure is facilitated by the use of Bewley's 'lattice diagram': a horizontal scale (of distance along a system in the direction of the initial surge) is combined with a downward vertical scale of time. A lattice of distance–time lines occupies the plane, the lines being sloped to correspond with the velocities of propagation in the system. The lines are marked with their surge voltage values in terms of v_1 and the various transmission and reflection coefficients.

The process is illustrated by the example in *Figure 2.33*. A 500 kV steep-fronted surge reaches the junction P of an overhead transmission line with a cable PQ, 1 km long. At Q the cable is connected to a second overhead line having a short circuit at S, distant 2 km from Q. The overhead lines have a propagation velocity of $u = 0.30$ km/µs and surge impedances z_0 respectively of 500 and 600 Ω; corresponding values for the cable are 0.20 km/µs and 70 Ω. Assuming the surge impedances to be purely resistive and neglecting attenuation, the Bewley lattice diagram is to be drawn for the system and the surge–voltage distribution found for an instant 19 µs after the surge wavefront reaches P.

Five sets of transmission and reflection factors are required, one at S and two each at P and Q for the two directions of propagation. The reflection factor is $\alpha = (z - z_0)/(z + z_0)$ and the transmission (or absorption) factor is $\beta = (1 + \alpha)$. Then

Direction left to right :
At P: $\alpha = (70 - 500)/(70 + 500) = -0.75$; $\beta = +0.25$
At Q: $\alpha = (600 - 70)/(600 + 70) = +0.79$; $\beta = +1.79$
At S: $\alpha = (0 - 600)/(0 + 600) = -1.0$; $\beta = 0$

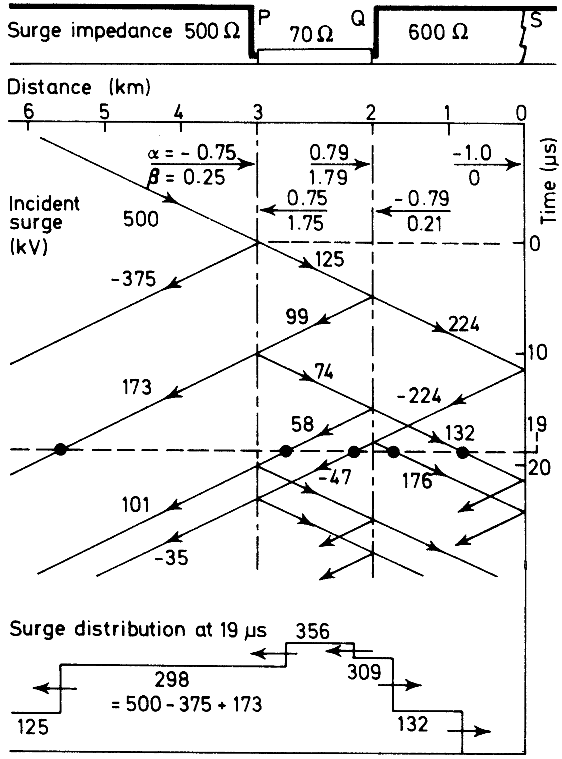


Figure 2.33 Bewley's lattice diagram

Direction right to left :
At P: $\alpha = (500 - 70)/(500 + 70) = +0.75$; $\beta = +1.75$
At Q: $\alpha = (70 - 600)/(70 + 600) = -0.79$; $\beta = +0.21$

The lattice diagram has distance plotted horizontally. The distance scale for the cable section PQ is enlarged by the factor 3/2 to take account of its lower propagation velocity. Time is plotted vertically with time zero at the instant that the surge voltage reaches P. Starting from the upper left-hand side, a sloping straight line is drawn to show the position at any instant of the surge wavefront. Reflection occurs at each junction, so that corresponding lines of the same slope (but running downward to the left) are drawn. The junctions are now marked with the appropriate voltages, using the calculated reflection and transmission factors α and β. Thus the incident surge at P is reflected with $\alpha = -0.75$ to give -375 kV; the transmitted voltage, with $\beta = +0.25$, is 125 kV. The former is turned back at P and is superimposed on the incident surge, while the latter proceeds in the direction PQ, to be split at Q into reflected and transmitted components. At any junction the marked voltages must sum on each side to the same total. The surge–voltage distribution for 19 µs is found by summing the voltages marked on the sloping lines up to this instant.

2.7 Electrical discharges

2.7.1 Introduction

Electric currents may be induced to flow through normally insulating materials by the formation of an electrical discharge. The discharge is maintained by the creation and movement of ions and electrons which in many discharges constitute a plasma (see Section 10.8). Such discharges may be produced between two electrodes which form part of an electrical network and across which a sufficient potential difference exists to ionise the insulation. Alternatively, discharges may be electromagnetically induced, for instance by strong radiofrequency fields.

Electrical discharges may be characterised for electrical network applications in terms of the current and voltage values needed for their occurrence (*Figure 2.34*).

2.7.2 Types of discharge

Discharges have historically been subdivided into two categories namely *self-sustaining* and *non-self-sustaining* discharges. The transition between the two forms (which constitutes the electrical breakdown of the gas) is sudden and occurs through the formation of a *spark*.

Non-self-sustaining discharges occur at relatively low currents ($\sim 10^{-8}$ A) (region 0A, *Figure 2.34*) of which *Townsend discharges* are a particular type. The form of the current–voltage characteristic in this region is governed firstly by a current increase caused by primary electrons ionising the gas by collision to produce secondary electrons, and subsequently by the positive ions formed in this process gaining sufficient energy to produce further ionisation. Such discharges may be induced by irradiating the gas in between two electrodes to produce the initial ionisation. They are non-sustaining because the current flow ceases as soon as the ionising radiation is removed.

When the voltage across the electrodes reaches a critical value V_s (*Figure 2.34*), current level $\sim 10^{-5}$ A, the current increases rapidly via a *spark* to form a self-sustaining discharge. The sparking potential V_s for ideal operating conditions (uniform electric field) varies with the product of gas pressure (p) and electrode separation (d) according to Paschen's law (*Figure 2.35*). There is a critical value of pd for which the breakdown voltage V_s is a minimum.

The self-sustaining discharge following breakdown may be either a *glow* or *arc* discharge (regions B–C and D–F, respectively, in *Figure 2.34*) depending on the discharge path and the nature of the connected electric circuit.

The region between V_s and B (*Figure 2.34*) is known as a '*normal*' *glow discharge* and is characterised by the potential

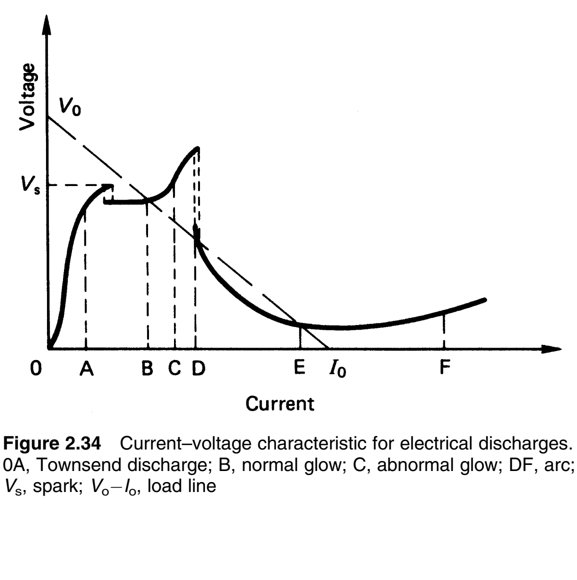


Figure 2.34 Current–voltage characteristic for electrical discharges. 0A, Townsend discharge; B, normal glow; C, abnormal glow; DF, arc; V_s, spark; $V_0 - I_0$, load line

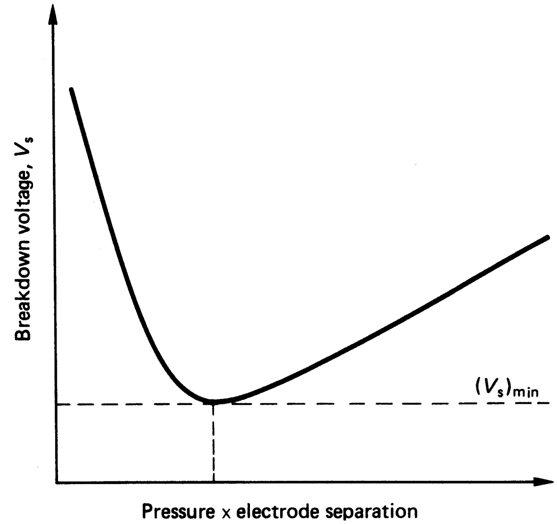


Figure 2.35 Breakdown voltage as a function of the pressure–electrode separation product (Paschen's law)

difference across the discharge being nearly independent of current, extending to at least 10^{-3} A if not several amperes. For higher currents the voltage increases to form the '*abnormal*' *glow discharge* (region C, *Figure 2.34*).

The glow discharge is manifest as a diffusely luminous plasma extending across the discharge volume but may consist of alternate light and dark regions extending from the cathode in the order: Aston dark space, cathode glow, cathode dark space, negative glow, Faraday dark space, positive glow (which is extensive in volume), the anode glow and the anode dark space (*Figure 2.36*). The glowing regions correspond to ionisation and excitation processes being particularly active and their occurrence and extent depends on the particular operating conditions.

The voltage across the glow discharge consists of two major components: the cathode fall and the positive column (*Figure 2.36a*). Most of the voltage drop occurs across the cathode fall.

For sharply curved surfaces (e.g. wires) and long electrode separations the gas near the surface breaks down at a voltage less than V_s to form a local glow discharge known as a *corona*.

The *electric arc* is a self-sustained discharge requiring only a low voltage for its sustenance and capable of causing currents from typically 10^{-1} A to above 10^5 A to flow.

A major difference between arc and glow discharges is that the current density at the cathode of the arc is greater than that at the glow cathode (*Figure 2.37*). The implication is that the electron emission process for the arc is different from that of the glow and is often thermionic in nature. At higher gas pressures both the anode and cathode of the arc may be at the boiling temperature of the electrode material. Materials having high boiling points (e.g. carbon and tungsten) have lower cathode current densities (ca. 5×10^2 A/cm^2) than materials with lower boiling points (e.g. copper and iron; ca. 5×10^3 A/cm^2).

The arc voltage is the sum of three distinct components (*Figure 2.36(b)*): the cathode and anode falls and the positive column. Cathode and anode fall voltages are each typically about 10 V. Short arcs are governed by the electrode fall regions, whereas longer arcs are dominated by the positive column.

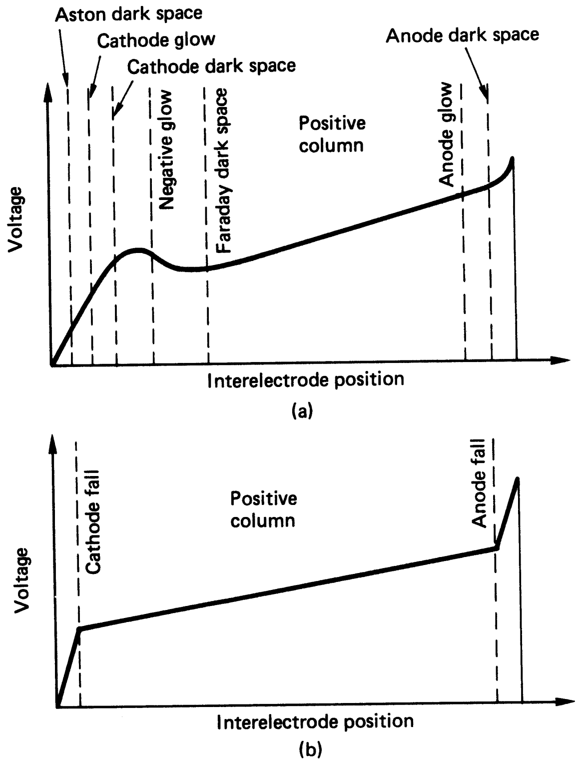


Figure 2.36 Voltage distributions between discharge electrodes: (a) glow; (b) arc

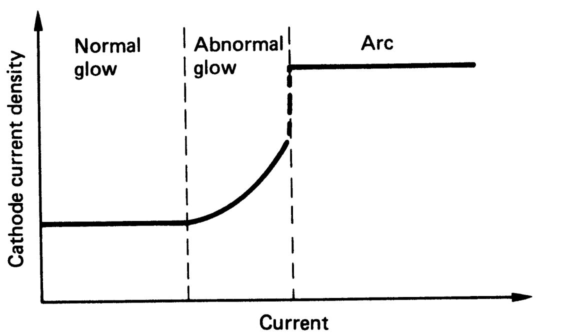


Figure 2.37 Cathode current density distinction between glow and arc discharges

At low pressures the arc may be luminously diffuse and the plasma is not in thermal equilibrium. The temperature of the gas atoms and ions is seldom more than a few hundred degrees Celsius whereas the temperature of the electrons may be as high as 4×10^4 K (*Figure 2.38*).

At atmospheric pressure and above, the arc discharge is manifest as a constricted, highly luminous core surrounded by a more diffusively luminous aureole. The arc plasma column is generally, although not exclusively, in thermal equilibrium.

The arc core is typically at temperatures in the range 5×10^3 to 30×10^3 K so that the gas is completely disso-

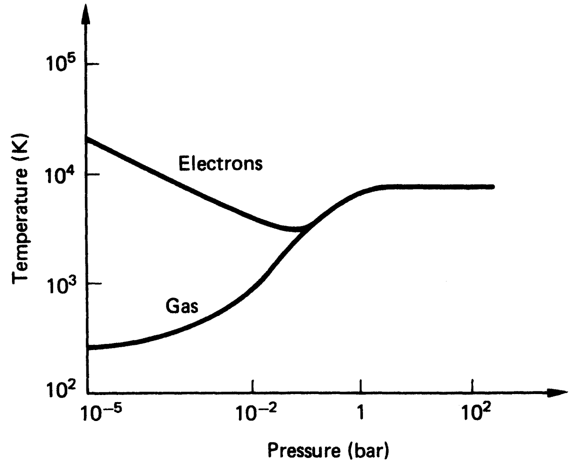


Figure 2.38 Typical electron and gas temperature variations with pressure for arc plasma

ciated and highly ionised. Conversely, the temperature of the aureole spans the range over which dissociation and chemical reactions occur (ca. 2×10^3 to 5×10^3 K).

The current voltage characteristic of the long electric arc is governed by the electric power (VI) dissipated in the arc column to overcome thermal losses. At lower current levels (10^{-1} to 10^2 A) the are column is governed by thermal conduction losses leading to a negative gradient for the current-voltage characteristic. At higher current levels radiation becomes the dominant loss mechanism yielding a positive gradient characteristic (*Figure 2.34*).

Thermal losses and hence electric power dissipation increase with arc length (e.g. longer electrode separation), gas pressure, convection (e.g. arcs in supersonic flows) and radiation (e.g. entrained metal vapours). As a result the arc voltage at a given current is also increased, causing the discharge characteristic (*Figure 2.34*) to be displaced parallel to the voltage axis.

2.7.3 Discharge–network interaction

For quasi-steady situations the interaction between an electrical discharge and the interconnected network is governed by the intersection of the load line V_oI_o (*Figure 2.34*)

$$V = V_o - IR$$

(where R is the series-network resistance and V_o is the source voltage) and the current–voltage characteristic of the discharge.

The negative gradient of the low current arc branch of the discharge characteristic produces a negative incremental resistance which makes operation at point D (*Figure 2.34*) unstable whereas operation at point E or B is stable.

In practice the operating point is determined by the manner in which the discharge is initiated. Initiation by electrode separation (e.g. circuit breaker contact opening) or by fuse rupture leads to arc operation at E. However, if the discharge is initiated by reducing the series resistance R gradually so that the load line is rotated about V_o, operation as a glow discharge at B may be maintained. If the cathode is heated to provide a large supply of electrons a transition from B to E may occur.

A variation of the source voltage V_o causes the points of intersection of the load line and discharge characteristic to vary so that new points of stability are produced. When the voltage falls below a value, which makes the load line tangential to the negative characteristic, the arc is, in principle, not sustainable. However, in practice, the thermal inertia of the arc plasma may maintain ionisation and so delay eventual arc extinction.

The behaviour of electric arcs in a.c. networks is governed by the competing effects of the thermal inertia of the arc column (due to the thermal capacity of the arc plasma) and the electrical inertia of the network (produced by circuit inductance and manifest as a phase difference between current and voltage). The current–voltage characteristic of the discharge changes from the quasi-steady (d.c.) form of *Figure 2.34* (corresponding to arc inertia being considerably less than the electrical inertia) via an intermediate form (when the thermal and electrical inertias are comparable) to an approximately resistive form (when the thermal inertia is considerably greater than the electrical inertia) (*Figure 2.39*).

2.7.4 Discharge applications

Electrical discharges occur in a number of engineering situations either as limiting or as essential operating features of systems and devices.

Spark discharges are used in applications which utilise their transitional nature. These include spark gaps for protecting equipment against high frequency, high voltage transients and as rapid acting make switches for high power test equipment or pulsed power applications. They are also used for spark erosion in machining materials to high tolerances.

Glow discharges are utilised in a variety of lamps, in gas lasers, in the processing of semiconductor materials and for the surface hardening of materials (e.g. nitriding). Operational problems in all cases involve maintaining the discharge against extinction during the low current part of the driving a.c. at one extreme and preventing transition

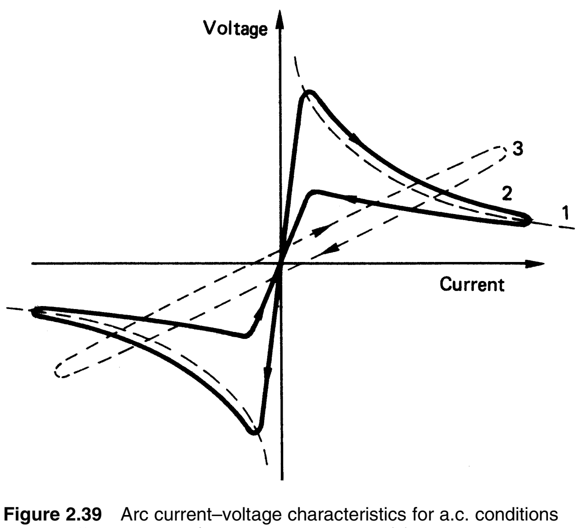


Figure 2.39 Arc current–voltage characteristics for a.c. conditions having different thermal/electrical inertia ratios: (1) thermal $\ll$ electrical inertia (d.c. case); (2) thermal $\simeq$ electrical inertia; (3) thermal $\gg$ electrical inertia (e.g. high frequency, resistive case)

into an arcing mode (which could lead to destructive thermal overload) at the other extreme.

Glow discharge lamps either rely on short discharge gaps in which all the light is produced from the negative glow covering the cathode (e.g. neon indicator lamps) or long discharge gaps in which all the light comes from the positive column confined in a long tube (e.g. neon advertising lights).

In materials processing the glow is used to provide the required active ionic species for surface treating the material which forms a cathodic electrode. Both etching of surface layers and deposition of complex layers can be achieved with important applications for the production of integrated circuits for the electronics industry. Metallic surfaces (e.g. titanium steel) may be hardened by nitriding in glow discharges.

Corona on high voltage transmission lines constitute a continuous power loss which for long-distance transmission may be substantial and economically undesirable. Furthermore, such corona can cause a deterioration of insulating materials through the combined action of the discharges (ion bombardment) and the effect of chemical compounds (e.g. ozone and nitrogen oxides) formed in the discharge on the surface.

Arc discharges are used in high pressure lamps, gas lasers, welding, and arc heaters and also occur in current-interruption devices. The distinction between the needs of the two classes of applications is that for lamps and heaters the arc needs to be stably sustained whereas for circuit interruption the arc needs to be extinguished in a controlled manner. The implication is that, when for the former applications an a.c. supply is used, the arc thermal inertia needs to be relatively long compared with the electrical inertia of the network (e.g. to minimise lamp flicker). For circuit-interruption applications the opposite is required in order to accelerate arc extinction and provide efficient current interruption. Such applications require that a number of different current waveforms (*Figure 2.40*) should be interruptable in a controlled manner. High voltage a.c. networks need to be interrupted as the current passes naturally through zero to avoid excessive transient voltages being produced by the inductive nature of such networks. Low voltage, domestic type networks benefit from interruption via the current limiting action of a rapidly lengthening arc. High voltage d.c. network interruption relies upon the arc producing controlled instabilities to force the current artificially to zero.

The arc discharges which are utilised for these applications are configured in a number of different ways. Some basic forms are shown in *Figure 2.41*. These may be divided into two major categories corresponding to the symmetry of the arc. Axisymmetrical arcs include those which are free burning vertically (so that symmetry is maintained by buoyancy forces) wall stabilised arcs, ablation stabilised arcs (which essentially represent arcs in fuses) and axial convection controlled arcs (which are used for both gas heating, welding and high voltage circuit interruption). Non-axisymmetric arcs include the crossflow arc, the linearly driven electromagnetic arc (which has potential for the electromagnetic drive of projectiles or by driving the arc into deionising plates for circuit interruption), the rotary driven electromagnetic arc (which may be configured either between ring electrodes and used for circuit interruption, or helically and used both for gas heating and circuit interruption) and the spiral arc with wall stabilisation.

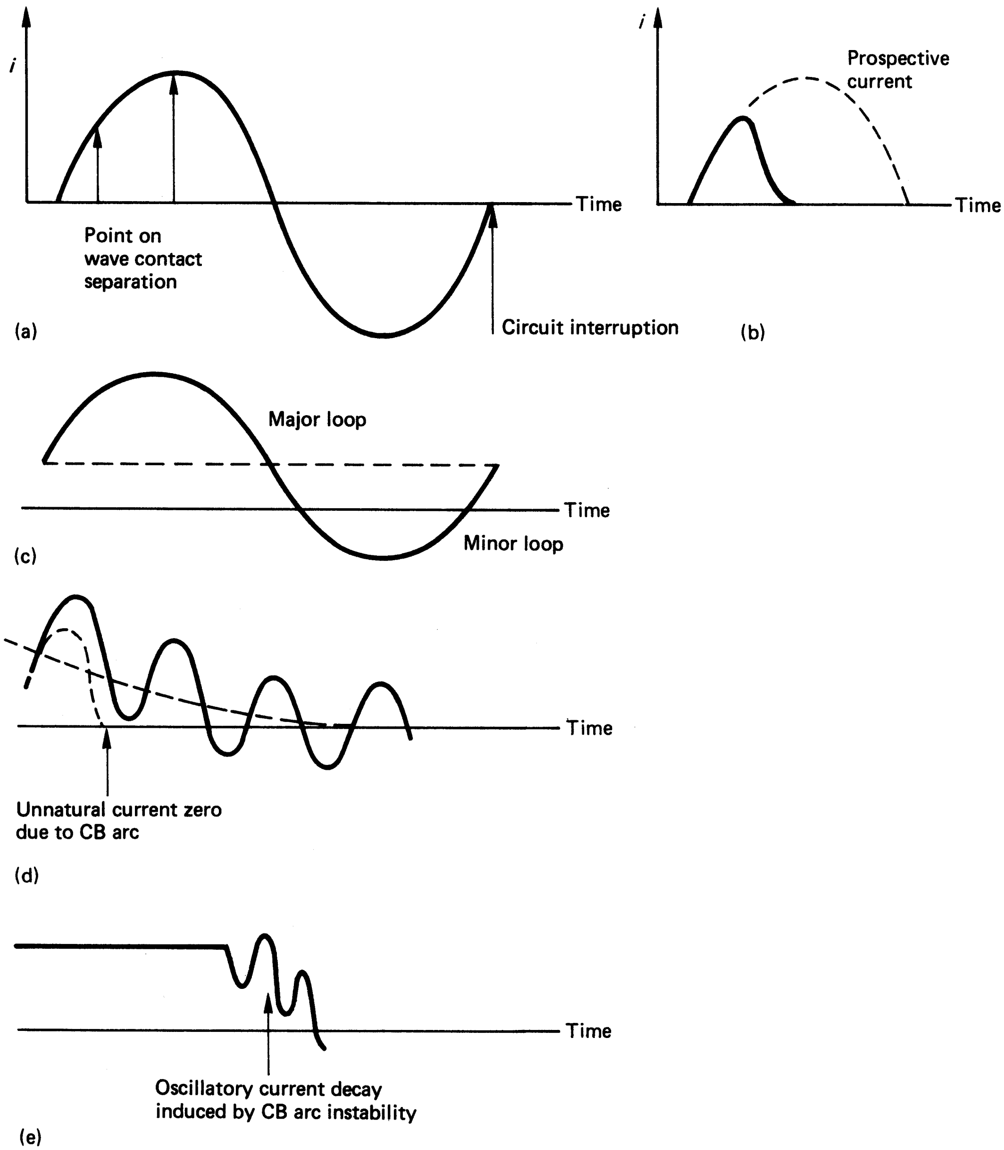


Figure 2.40 Circuit breaker (CB) current waveforms: (a) a.c. arc interruption; (b) current limiting effect (domestic CB); (c) asymmetric waveform; (d) generator fault waveform; (e) d.c. arc interruption

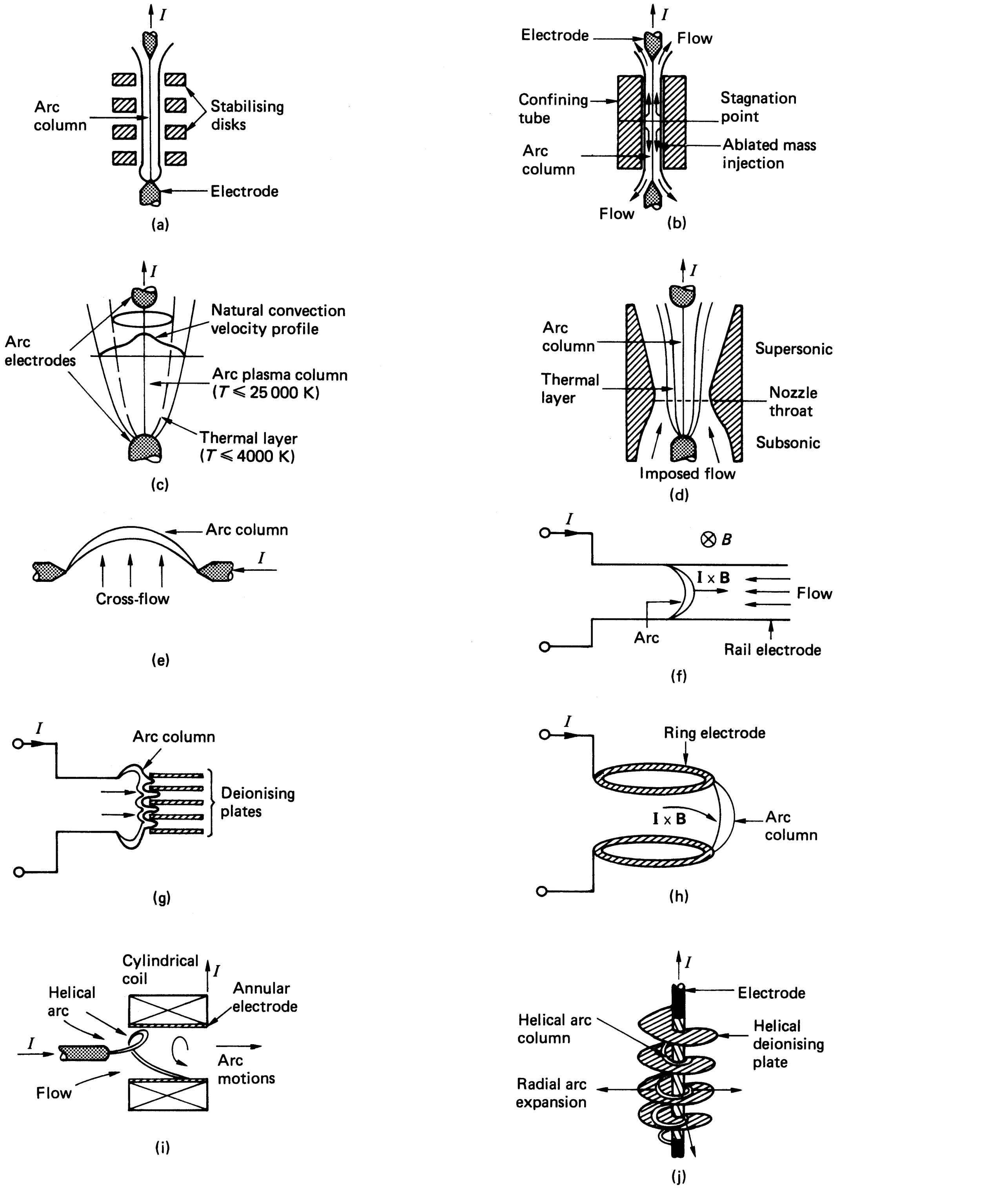


Figure 2.41 Different arc types: (a) wall stabilised; (b) ablation stabilised; (c) free burning (vertical); (d) axial convection; (e) cross flow; (f) linear electromagnetic acceleration; (g) deionising plates; (h) rotating arc (ring electrodes); (i) helical arc; (j) spiral arc with deionising plate